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Fibrations, isotopies and cell structures on arrangements

What is the topology of $M = M(\mathcal{A})$ = complement of $\mathcal{A}$ ("Static")

How does topology depend on coefficients. ("dynamic")

$$\alpha_i(z_1, \ldots, z_\ell) = a_{i_1} z_1 + \cdots + a_{i\ell} z_\ell$$

gives matrix R $(n \times \ell)$

e.g. $\mathcal{B}$): Boolean $R = Id$ $Q = z_1 \cdots z_\ell$

$$M = \prod^{\ell} (\mathbb{C} - 0) \underset{\text{diffeo.}}{\cong} \prod (S^1 \times \mathbb{R}^1) \cong T^\ell$$

$\mathcal{A}$) Artin / Braid arrangement

$\pi_1(M)$ = pure braid group

$\mathcal{H}$) R random get generic arrangement

These three examples are Instructive / Important / Motivating.

$\mathcal{A}_{\text{braid}}$ $\qquad \mathcal{B} \qquad \mathcal{H}$

$$
\begin{tikzcd}
 & & \mathbb{C}^* \arrow[d] & \\
h \circlearrowright Q^{-1}(1) = F \arrow[rr] \arrow[drr, "p"] & & M \subset \mathbb{C}^{\ell} \arrow[r, "Q"] \arrow[d, "\text{Hopf map}"] & \mathbb{C}^* \\
 & & M^* \subset \mathbb{P}^{\ell-1} &
\end{tikzcd}
$$

Milnor fiber

Milnor fibration

Zariski

n-fold cover

h is the covering transformation
monodromy

<u>Thm</u> (i) $M \cong M^* \times \mathbb{C}^*$ (except T = empty arrangement)

(ii) $\pi_k(M) \cong \pi_k(M^*) \cong \pi_k(F)$, $k \geq 2$

(iii) $\pi_1(M) \cong \pi_1(M^*) \times \mathbb{Z}$

$$\frac{\pi_1(M^*)}{p_* \pi_1(F)} \cong \mathbb{Z}/n\mathbb{Z}$$

<u>$K(\pi,1)$-problem</u> When is M aspherical?

Instead ask:

How much is M like a torus?

(i) Arnold, Brieskorn, Orlik-Solomon ('68-'80)

$\wedge H^1(M) \longrightarrow H^*(M)$ onto.

(ii) $\Phi : \pi_k(M) \longrightarrow H_k(M)$ the Hurewicz homomorphism for $k>1$ is the zero map

$H_2(\pi_1(M)) \cong H_2(M)$

(iii) M has a CW-structure with # of k-cells $= b_k(M)$ (Dunca-P, R)

<u>Thm</u> (Papadima-Suciu, D-P, R)

M is $K(\pi,1)$ iff $\pi_1(M)$ minimal and

$$b_k(M) = b_k(\pi_1(M))$$

<u>Aside</u>: How much topology does the lattice have? or encode

i) Lattice $\Rightarrow H^*(M)$

ii) $L \not\Rightarrow \pi_1(M)$ Ryb.

Milnor fiber (is an extremely corse invariant of the fundamental gp)

F

Is $H_* F$, $\pi_1(F)$ determined by lattice?

Torsion $H_*(F)$

Dynamic topology

let coeff. vary

treat $\mathcal{L}$ as R (an $n \times \ell$ matrix)

Define "Moduli spaces"

(i) $R \sim R'$ iff $\exists$ nonsingular diag. S
Left equivalence so that $SR = R'$

$$\mathcal{M}_{\ell,n} \cong (\mathbb{CP}^{\ell-1})^n$$

(ii) S as above and nonsingular T such that
Left-Right equivalence $R' = SRT$

Equivalence class $\mathcal{R}_{\ell,n}$
Realization space

Note: Topology of $\mathcal{M}$ or $\mathcal{R}$ is complex and full of traps for the incautious.

Thm Topology is constant along lattice isotopy.

Questions

i) Topology of "strata"

properties of stratification

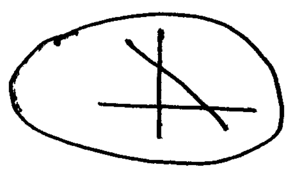
⋮ special classes

ii) Deletion maps ; properties

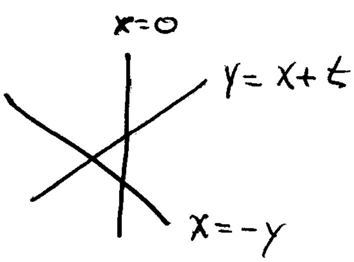
Rep'n (LKB)

iii) Finite-type invariants ; Jones polynomial

If we take out the 3-cell in $T^3 = S^1 \times S^1 \times S^1$

we get 

Homework



M^*

1	# 2-cells
2	# 1-cells
1	# 0-cells

Draw the CW-complex

$T^2 \cong F$

3	2-0 cells
6	1-0 cells
3	0 - cells