Problem Session

Michael Falk - conducts session

Uli Walther

Fact: $f \in \mathbb{C}[x_1, \dots, x_n] = R_n$

Let D_n be the n-Weyl algebra

$$D_n = \mathbb{C}[x_1, \dots, x_n, \partial_1, \dots, \partial_n]$$

where $\quad \underset{\substack{\Uparrow \\ R_n}}{\partial_i(g)} = \partial/\partial x_i \, (g)$

$$\Rightarrow \quad x_i \partial_i + 1 = \partial_i x_i .$$

Now, the fact: $\exists \; P(s) \in D_n[s]$

such that $\quad P(s) \cdot f^{s+1} = b_f(s) \cdot f^s$

$b_f(s) \in \mathbb{C}[s]$ and the collection of all $b(s)$ that appear in such an equation form an ideal in $\mathbb{C}[s]$, generated by $b_f(s)$.

Q: Compute $b_f(s)$ if $f = Q_{\mathcal{A}}$

Properties of $b_f(s)$:

$b_f(s) \in \mathbb{Q}[s]$, with all roots rational, negative, and a multiple of $s+1$.

$b_f(s)$ is related to

- Jacobian ideal
- Milnor fiber
- log-canonical threshold
- asymptotic behavior of

$$\int_{\substack{\text{near}\\ \text{origin}}} |f|^s \, dx \qquad \begin{array}{l} s \in \mathbb{C} \\ \text{assume} \\ f(0) = 0 \end{array}$$

Milnor fiber suppose f is homogeneous let $d_1, \dots, d_k$ be the degrees (as differential forms) of homogeneous generators of $H^{top}_{dR}(f^{-1}(1), \mathbb{C})$

Then $b_f\left(-\left(\frac{n+d_i}{\deg(f)}\right)\right) = 0$

Suppose f is an isolated singularity, R_n, $(f_1, \dots, f_n)$ the Jacobian

then $\deg\left(\frac{R_n}{(f_1,\dots,f_n)}\right) \longleftrightarrow$ roots of $\frac{b_f(s)}{s+1}$

$b_f(s)$ is local:

Let $p \in \mathbb{C}^n$

define

$b_f^p(s) :=$ the generator of the ideal $\{b(s) \mid \exists P(s) f^{s+1} = b(s) f^s$ with $P(s) \in (D_n[s])_p\}$

Then $b_f(s) = \text{l.c.m.}\left(b_f^p(s) \mid p \in \mathbb{C}^n\right)$

<u>Fact</u>: f = homogeneous in 2 variables factoring completely, $\deg(f) = d$

$\Rightarrow \quad b_f(s) = (s+1)(s+\frac{2}{d})(s+\frac{3}{d}) \cdots (s+\frac{2d-2}{d})$

<u>Also</u> if f is a generic central arr. of deg d in n variables

$$b_f(s) = (s+1)^{n-1}\left(s+\frac{n}{d}\right)\left(s+\frac{n+1}{d}\right)\cdots\left(s+\frac{2d-n-2}{d}\right)$$

<u>Reference</u>: www.math.purdue.edu/~walther
↳ Research

look at Bernstein
Yano (78)
Malgrange (73-77)
Kashiwara (76-78)
on the above website

Richard Randell

Problems on the Fundamental Group

① Does $\pi_1 M$ have torsion? (Is $\pi_1(M)$ torsion free?)

Pro
true if M is $K(\pi,1)$
nice presentation

Con
π_1 not nice (X_3)
not nice presentation

② Is $H_1(F)$ torsion-free?

Pro
true for general position

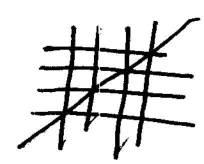
Con
Cohen, Denham, Suciu have counterexample for multiarrangement

③ Is $\pi_1(M)$ linear (i.e. a faithful f.d. represen.)?

Lawrence Krammer, Bigelow,

take cover ~~of~~ corresponding to M_B, E_G the map

$$\begin{array}{ccc} E & \to & M^{\ell+2} \\ & & \downarrow \\ & & M^{\ell}_B \end{array}$$



$\pi_1(E) \to \mathbb{Z}\oplus\mathbb{Z} = G$

$\pi_1(M_B)$ acts on $H_2(E_G, \mathbb{Z})$

Paris - Paoluzzi

(4) $\pi_1(M)$ residually nilpotent
(finite)

Michael Falk

Problem (1) Find a configuration of 30 points in $\mathbb{P}^2$ $\mathcal{C} = \mathcal{C}_1 \dot\cup \cdots \dot\cup \mathcal{C}_5$, $|\mathcal{C}_i| = 6$

a "5-net" such that every non-trivial* line consists of one point from each $\mathcal{C}_i$

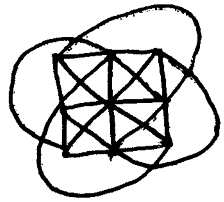
* more than 2 points

or (2) Find a central 3-arrangement
$\mathcal{A} = \mathcal{A}_1 \dot\cup \cdots \dot\cup \mathcal{A}_5$
such that every multiple point (mult > 2) has exactly one line from each class.

(3) Find a pencil of sextic plane curves w/ 5 singular elements, each a union of lines.

Context: Which 3-arrangements support resonant weights?

Ex



a "4-net", the only one known!

(4) Calculate the elementary divisors (or rank over $\mathbb{Z}_p$)

of point-line incidence matrices

of point configurations in $\mathbb{P}^2$

(or arrangements in $\mathbb{P}^2$)

$$\begin{array}{cc} & 1 \; \cdots \; n \\ \text{lines} & \begin{bmatrix} \text{incidence} \\ \text{matrix} \end{bmatrix} \end{array}$$

Alex Suciu

can compute a large amount of arrangement calculations with

GAP
Macaulay 2 — "Arrangements" package

Dean Serenevy

information on how to use the Arrangements package dean.serenevy.net/

and can use this on MSRI

linux terminal

_gap4r4

LoadPackage("ng");

LoadPackage("Arrangement");

if there are requests email:

dean@serenevy.net

and website has manual