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9/mar Simplicity & Disjointness

Isomorphism

$(X, \mathcal{A}, m, T)$ & $(Y, \mathcal{B}, \mu, S)$

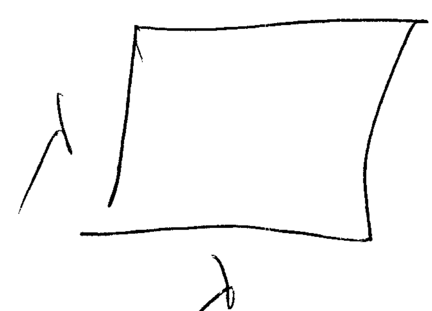
$$\begin{array}{ccc} X & \xrightarrow{\Phi} & Y \\ T \downarrow & & \downarrow S \\ X & \xrightarrow{\Phi} & Y \end{array}$$

JOININGS: $X\ T \quad Y\ S \quad \lambda$

$X \times Y$ & λ is $T \times S$ invariant

$\lambda|_{\mathcal{A}\times Y}$ is m V

$\lambda|_{X \times \mathcal{B}}$ is μ H



T & S isomorphic $\Longleftrightarrow \exists \lambda$ s.t.

$$V = H(\lambda)$$

DISJOINTNESS

Only joining is a product

Rudolph: T is weakly mixing

$T \times T$ λ $\mu \otimes \mu$

Also diagonal joinings

Family of joinings

Veech: Simplicity

T is simple if

$T \times T$ λ

either λ is a product or

$\mathcal{V}$ and $\mathcal{H}$ are identical mod λ

$S^1 = \mathbb{R}/\mathbb{Z}$ λ T_2 & T_3

$\mathbb{Z}^2$ natural extension

$T_2 T_3$ is simple $\Leftrightarrow$ There are no continuous invariant measures other than λ

All horocycle flows are factors of simplex

Assume T has H-property

λ $T \times T$, then $T \times T$ is a finite extension of its marginals

<u>Definition</u> $(X, \mathcal{A}, m, T)$ and group G/H

$X \xrightarrow{\Phi} G$

G/H $\lambda \times X$ is $T_\Phi(x, g) = (Tx, \Phi(x) g)$

$(G, \mathcal{B}\mu, S)$ is distal above $(X, \mathcal{A}, m, T)$

~~Lemma~~: <u>Quasi-simple</u>:

$(X, \mathcal{A}, m, T)$ is quasi-simple

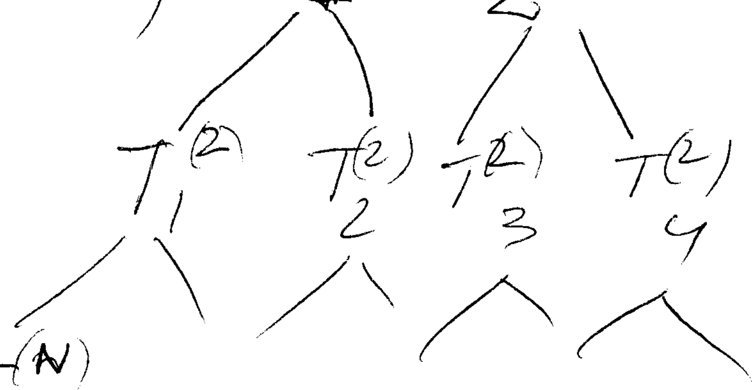
λ $(X)(X)$ λ is a product

or

$T \times T$ is a distal extension of its marginals

Infinitely divisible Transformations

$(X, \mathcal{A}, m, T) = T_1^{(1)} \times T_2^{(1)}$



Nth Stage $= \prod_{k=1}^{n^2} T_k^{(N)}$

Ex Bernoulli Transformation
Gaussian Transformation
Poisson Suspensions

T is quasi-simple, then T is disjoint from ∞-divisible & imbeddable in a flow

Lemma: T is quasi-simple

$T \times S1 \times S1$, S^1 is imbeddable in a flow

$\Rightarrow$ It's a product