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Homology of resonant local systems

all the considered local systems are rank 1 on complements of arrangements

$\mathcal{A}$ arr. of complex hyperplanes in $\mathbb{P}^m$ s.t.

$\bigcap_{H\in\mathcal{A}} H$ empty. $M = \mathbb{P}^m \setminus \bigcup_{H\in\mathcal{A}} H$ is complement

$N = \bigcup_{H\in\mathcal{A}} H$ divisor in $\mathbb{P}^m$

$\mathcal{L}_\tau$ rank 1 local system on M

$$\tau : \pi_1 M \longrightarrow \mathbb{C}^*$$
$$g_H \longmapsto \tau_H \in \mathbb{C}^*$$

__definition:__ $\mathcal{L}$ __generic__ if $H_k(M,\mathcal{L}) = 0$

$\forall\, k \neq m$; $\dim_{\mathbb{C}} H_m(M,\mathcal{L}) = |\chi(M)|$

$X = H_{i_1} \cap \dots \cap H_{i_r}$ is a

__dense edge__

__Def__ $\mathcal{L}_\tau$ is __resonant__ at dense edge X if if $\tau_X = 1$ monodromy coefficient

Also, $\mathcal{L}$ is <u>resonant</u> if resonant at some dense edge.

Canonical embedded resolution N in $\mathbb{P}^m$

$p: Z \to \mathbb{P}^n$

$D = p^{-1}(N)$

normal crossing divisor

$p: Z \setminus D \xrightarrow{\sim} M$ diffeomorphism

irreducible smooth components D_X, X dense edge of $\mathcal{A}$,

$\tau_X =$ monodromy of loop about D_X

<u>Thm</u> If $\mathcal{L}$ is ~~x~~ non-resonant and generic then $h: H_m(M, \mathcal{L}) \to H_m^{lf}(M, \mathcal{L})$ is an isomorphism.

locally finite homology

[K, S.T.V., F.T., O.S, V, ...]

<u>Remark</u>: [Cohen-Dimca-Orlik] If $\exists H$ hyperplane s.t. $\forall$ dense edges $X \supseteq H$ $\tau_X \neq 1$ then $\mathcal{L}$ is generic.

Main Problem: Study the homomorphism h when $\mathcal{L}$ is resonant generic.

If $\mathcal{L}$ is resonant then, in general, h fails to be an isomorphism.

i) Want $\operatorname{Ker} h$, $\operatorname{Im} h$ = Regulizable classes and their dimensions

$$= H_m^{lf}(M, \mathcal{L})_{\underline{reg}}$$

ii) Want nice, natural (geometric) bases for

$$H_m(M, \mathcal{L}), \; H_m^{lf}(M, \mathcal{L})_{\underline{reg}},$$
$$H_m^{lf}(M, \mathcal{L}).$$

Known $m=2$

Mimachi, Ochiai, Yoshida

$$\dim_{\mathbb{C}} H_m^{lf}(M, \mathcal{L})_{\underline{reg}}.$$

$\mathcal{A}$ real complexified arrangement

$\mathcal{L}$ non-resonant $H_m(M, \mathcal{L}) \xrightarrow[\simeq]{h} H_m^{lf}(M, \mathcal{L}) =$

with Bubc basis

$$= \bigoplus_{\substack{\Delta \in \text{bounded chambers} \\ \text{in } \mathcal{H}_{\mathbb{R}}}} [\Delta] .$$

For $\mathcal{L}$ resonant, in general, $H_m^{lf}(M, \mathcal{L})$ is generated by $\{[\Delta]\}_{\Delta \in \text{bdd chambers}}$ but there are relations among them.

Theorem 1 Let Δ be a bounded chamber in $\mathcal{H}_{\mathbb{R}}$ s.t. $\forall X \subseteq H$ (dense edges), $\forall H$ touch $\overline{\Delta}$

then $[\Delta] \in H_m^{lf}(M, \mathcal{L})_{\text{reg}}$

s.t. $\mathcal{L}$ is non-resonant at X.

Proof Uses ideas of Aomoto, Tsuchiya & Kanie

Technique: controlled tubular nbhd stratified set $\overline{\Delta}$

Remark: Vassiliev - general construction for hypersurfaces

and other work by Orlik & Silvotti

$\mathcal{A}$ fiber type arrangement

$$M_{\mathcal{A}} \text{ is a } K(\pi_1 M, 1)$$

$$\pi_1 M = \mathbb{F}_{d_m} \rtimes \cdots \rtimes \mathbb{F}_{d_1}$$

iterated semi-direct product of free grps

D. Cohen Pure braid monodromies

Cohen-Suciu Constructed a finite resolution of $\mathbb{Z}$ of free modules over $\mathbb{Z}\pi$ for $\pi = \pi_1 M_{\mathcal{A}}$

Using these ideas:

Theorem Z: $\mathcal{A}$ fiber type arrangement
$\exists$ finite cellular chain complex of $\mathbb{Z}\pi$-mods which is homotopy equivalent with cellular chain complex $C_*^{lk, proj}(\tilde{M})$

$\tilde{M}$ - universal covering

$$\cdots \to C^{lf}_{m+1} \to \underset{\textstyle Z^{lf}_m}{\overset{\shortparallel}{C^{lf}_m}} \to 0$$

where $C^{lf}_{2m-k} = \bigoplus_{1 \le p_1 \le \cdots \le p_k \le m} (\mathbb{Z}G)^{d_{p_1} \cdots d_{p_k}}$

with twisted differential
(essentially given by version of Gassner
representation)