Federico Ardila

Bergman complexes and Coxeter Arrangements

(work w/ Reiner, Williams)

Outline

1. Amoebas and the Bergman complex
2. Berg. cmplx
3. positive Berg. cmplx
4. see slide

Amoebas: X cmplx variety

$$\mathcal{A}(X) = \mathrm{Log}\, X = \{\log|z_i|\} \quad \text{solutions}$$

Amoebas: appear in many places, start of tropical geometry, hard to understand
So,
Discribe Bergman complex, put sphere around Amoeba.

Thm: If X is d-dim. & irred.
B(X) is pure (d-1)-dim.

Thm

$B(X) = \{ w \in S^{n-1} \mid \mathrm{in}_w(I) \text{ contains no monomials} \}$

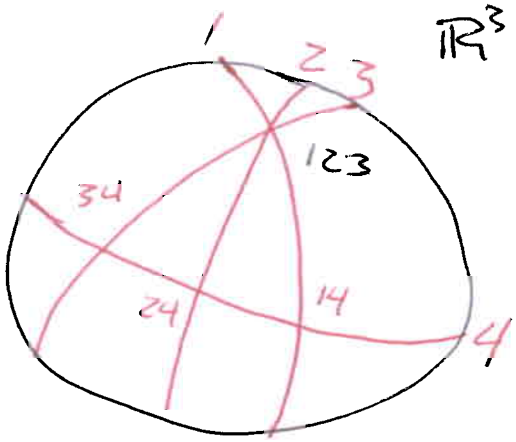
There is a matroid M_X associated to a linear subspace of $\mathbb{C}^n$, X.

Can define the Bergman complex of a matroid. $B(M)$.

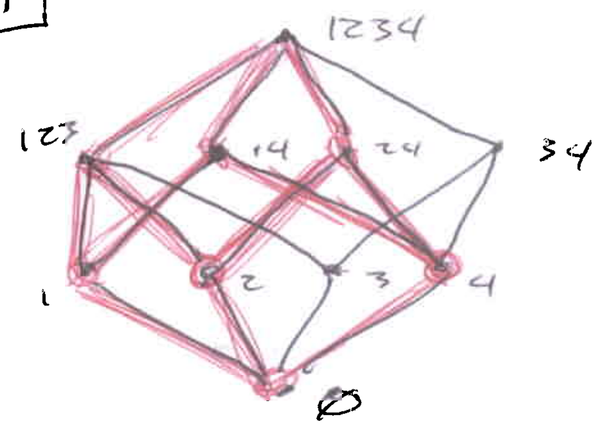
Goal: Describe $B(M)$ topologically, combinatorially. (connected?)

Think of M as the matroid of an arr.

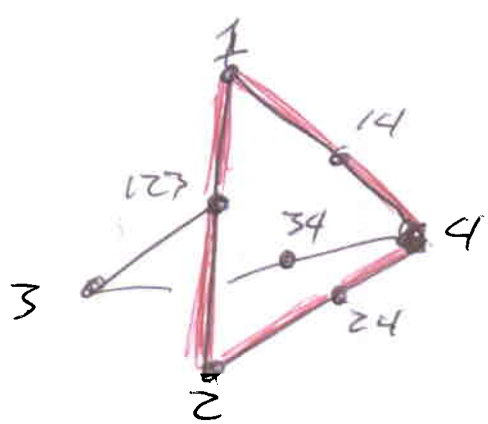
ex:



L_M



$\Delta(I_M)$



The possitive part of $B(M)$. Oriented Matroid.

So, in hyp. arr's we specify a region,

Motivation:

$$B(M) \longleftrightarrow \text{tropical variety}$$

$$B^+(M) = \{ w \in \underbrace{S^{n-2}}_{\text{sphere}} \mid \text{each } C \in \underbrace{\mathcal{C}}_{\text{circuits}} \text{ achieves its } w\text{-max in } C^+ \text{ and } C^- \}$$

There is a nice discription. Uses Las Vergnas face lattice of M.

ex: red part in above lattice

Thm: [A, k, w] with certain conditions of M. The pos. Bergman complex is the order complex of the Las Vergnas face lattice.

Coxeter arr's

Φ root system
M_Φ be the matroid

<u>Goal</u>: describe $B^+(M_\Phi)$ and $B(M_\Phi)$

acyclic orientation of M_Φ

$\Updownarrow$

region of real $\mathcal{A}_\Phi$

$\Updownarrow$

choices of roots

A <u>tube</u> in a Dynkin diagram is a connected subgraph of a Dy. diag.

A <u>tubing</u> is a collection of compatible tubes. (all site inside)

Have a poset of tubing.

<u>Thm</u> $B^+(M_\Phi)$ is dual to the associahedron graph.

The nested set complex encodes the combinatorics of this wonderful model.