

Olympiad Training and Math Camps  
Chair: Titu Andreescu  
12/18/04 9-11:30AM

Speakers:

Tom Roby (Ross Mathematics Program, ProMYS)  
David Kelly (HCSSiM)  
Rick Sommer (SUMaC)  
Mira Bernstein (Canada/USA Mathcamp)  
Zuming Feng (MOSP)  
Abigail Thompson (COSMOS)

Tom Roby (Ross Mathematics Program, ProMYS)

We all do this as a labor of love, not for quick money. In fact, most of us volunteer. I'll also talk about the Hampshire College Summer Studies in Mathematics program.

I started at the Ross program as a student in 79 and a junior counselor in 80, then with a dearth of head counselors, I shared the position in 81. As a graduate student, I got invited to be senior staff at the Hampshire program. There has always been a friendly rivalry between Hampshire and Ross and it was great to be able to see both of the programs. I was the founding head counselor of ProMYS in 89 and stayed with that through 91 until I went to Japan. I returned to ProMYS in 2000 to work with the teacher program, and then went to California to do K-12 outreach while there was a good amount of money for that.

Arnold Ross started the program when he was 50 and taught for over 40 years in the program. Everything I do, including my work with teachers was shaped by what I learned in the Ross program. Arnold Ross came from the Russian tradition. Another reason to thank the Russian tradition was that Sputnik inspired the government to put money into things like this. The program is 8 weeks and multilevel, and the NSF has not liked to fund multilevel programs, they like programs you go through once and then you are done. But multilevel is crucial because it becomes a peer mentoring environment. Ross's approach was hands-off and he trusted the counselors to take care of the students.

The problem sets are exploratory and the lectures are very open ended. If you just came to one lecture, it would seem like he wasn't teaching anything because he lectures were focused around the problem sets.

There are two midterms and a final and the scores are private so as not to be competitive. Ross invited the world's expert in p-adic analysis to come teach an advanced course and ran a parallel course in real analysis so that students could see the differences. No one should have abstract algebra before they've had a course in number theory.

Ross used to joke what he did was "Moore" method, but that meant E.H. Moore. The method was students trying to solve the problems. There was large amounts of funding from NSF in early years but now it is harder to get funding.

Two former counselors at Ross started the ProMYS program. The program is similar except that it is 6 weeks long and the problem sets are just 6 weeks of the Ross problem sets. Glenn Stevens thought he should be able to make some changes to the problem sets, but as he changed one thing, he would realize it had a huge impact on the rest of the problem sets. The difficulties of getting started include needing counselors, who would usually be alums, but that's a problem if you are starting a new program. If you haven't been through the problem sets it is hard to know how things connect up.

HCSSiM was founded by (David) Kelly, and is more time structured. Ross is structured around problems, and there is little scheduled class time. Students can stay up until 3 in the morning working on problem sets. The Hampshire program has 7 hours of scheduled classes and problem sessions a day. There are no exams or grades but students and faculty write (self-)evaluations. The goal is to reduce the amount of competitiveness as much as possible. While Ross is less competitive than an Olympiad program, there still is a streak of competitiveness. But Kelly is really trying to make sure mathematics is inclusive and relaxed.

Compared to contests, these programs provide peer group support for students who might be the only good math student in their school, provide much need challenge, and bring students into mathematics and problem solving.

Contrasted to contests, the competitive aspect is de-emphasized, and students go deeper into mathematical contest (proving quadratic reciprocity on their own), much more time for exploration is allowed, helps bridge the gap from doing (5 sec, min, hour) problems to doing research, collaboration is encouraged. Joe Gallian said "suppose you've co-authored 2 papers with 2 other authors and 1 paper with one other author, how many papers have you written?" "Three!" While we all have stereotypes that mathematics is working for 7 years alone in your attic, we all know that mathematics requires collaboration these days. Please encourage your students to come, but make sure they understand the difference is between these programs.

Perennial Issues: diversity is always an issue, charges of elitism—it is really important when you are applying for funding to respond to these charges. We need reasonable arguments that this isn't an elitist thing. How can we collaborate to have better public relations? We shouldn't all be competing for the same funds. We need to be increasing the pool of funding through public relations.

Question: How do you answer charges of elitism? We are trying to reach out and find talent wherever it is. We do have a lot of special programs in place for people who are struggling. We need to help groups that are struggling with school for other reasons, i.e. they are bored.

Comment: Nobody asks if baseball camps are elitist. 90% of the kids won't become professional baseball players.

Comment: These students are special and have special needs. The education system is failing them.

Comment: The difference with the baseball camps is that they aren't asking for public funding.

Comment: These programs are elitist, and we need to admit that and explain why it is good and should be publicly funded. We need to work more of finding talent inclusively.

Comment: (Mark Saul, NSF) It is important to distinguish charges of elitism and charges of underrepresentation. As a community we need to work on the underrepresentation. There is another charge of elitism—that is isn't good for the child; we need to respond to this and show that when kids are with their peers they come out of their shells. Even in education, that message is not gotten out.

Comment: There are some very talented students who are inexperienced in mathematics. A lot of these programs look to students who have mathematical abilities because they have had experiences to develop them. Elitism isn't bad if it is democratic elitism. When you go to the Ross program, you are among students that are fairly well off. If you are going to dip into other talent bases, you need to be prepared for that, but if you have only one Hispanic student, you need to think about how to deal with the social issues there. You may need more than just one, and you may need a counselor who is sensitive to that.

Comment: I think sports are as important as math. As the Greeks, we have to be good in mind as well as body. If we want to market these programs, it is worth including sports activities in them.

David Kelly (HCSSiM)

The post-Sputnik era led to the initial funding of the program. Our jobs as teachers and as mathematicians have been changed by computers. I find myself reminding my students how different mathematics is with computers. Tom Banchoff in the late 70s has just made a film on the hypercube, and came to show it to the students. At lunch he mentioned he has some film on the complex exponential function, which he showed on 16mm film. I had already taught complex analysis and knew how important it was. To see a picture of it was quite a thing.

I was involved in summer programs for 10 years before coming to Hampshire.

When I was first teaching, I got a teletype connection to a computer for my students. Their exercise was to keep doubling the number of sides in a polygon inscribed in a circle to estimate pi. Students got numbers that got greater than pi because of a roundoff error, but he joked that it was the "Gauss effect."

In many years of the program, we have talked about fractional linear transformations and the invariance of the cross ratio. To prove that the cross ratio is invariant, there is a mechanical way, just replace each  $z$  by  $(az+b)/(cz+d)$ , a more elegant way is to just recognize that each fractional linear transformation is a composition simpler functions you can easily check, or more elegantly realize that the cross ratio is a fractional linear

transformation itself. A student did the first method immediately by plugging the expression into Mathematica.

Let's evaluate programs:

Broad Range of Mathematical Activities (solving problems, forming problems, exploring, making definitions, writing, board work, generalizing, etc.) We try to include all of these in Hampshire including reading mathematical papers, talking to professional mathematicians.

Residential nature. The Hampshire program has the entire faculty live in the dorms with the students, eat meals, do recreation. This gives the students a better idea of what it is to be a mathematician. It is hard to get a family to participate, usually a spouse surrenders their spouse to the program for the summer. But occasionally we are able to bring a family to the program.

Assets, alums, anecdotes. The big asset is the alums. They give you tremendous gratification in terms of "have I done anything good?" When the NSF most recently pulled out of funding, I had to become a fundraiser. This forced me to write to 25 years of alums, while we got money, what was really gratifying was to hear from alums I hadn't heard from for 25 years saying how this program was important to them.

Community, collaboration. There are all sorts of communities: IMO, Putnam, REUs, Budapest Semesters, Project NEXT. The yellow pig has become the emblem of the summer studies.

Hours. We have all this time, there is no rush. We can go down dead ends. I was a reviewer for NSF summer programs and was amazed at how many people don't mention what students do in off hours. We've worked out a great program of having free time in the afternoons and problem sessions in the evenings. It is harder for students to get in trouble in the afternoons. We have a great number of contact hours and a five to 1 faculty to student ratio.

Impact. The impact is greatest on individual students, but also has an impact on their communities back at school. The NSF used to require students to give a talk when they went back to their high schools. This was a great thing! The students had to think about what they were going to talk about, and then could spread their enthusiasm and also teach their teachers something. We have a debriefing session at the end of the program to give students resources to continue in mathematics and deal with things back at their schools. We want to make sure they don't go back to their schools with arrogance.

Spreading the word, selection, serving. We choose our students, faculty, curriculum. We have to realize that a teacher that can do anything outside of what they are demanded to do is amazing. It is important that we spread the word and shouldn't rely on teachers or expect them to know everything. It is important to serve the students. Minorities and women need to sometimes see similar teachers. We need to give these groups special attention.

Teacher training. A lot of people have got inspired into teaching from the program. A lot of people who teach in the program were students in the program.

Ownership. Students take ownership of problems and ideas. They name conjectures after the people that made them. At the end of the program we can write a very detailed recommendation of their work. It is the most incisive, detailed evaluation they are ever going to get. The students also write a self-evaluation. This makes students feel responsibility for their own education. We guide them, but give them a sense of empowerment. One student said in his final evaluation "I am proud to have made more wrong conjectures than anyone else.

Question: Has the non-competitive nature of the program changed, going from the countercultural 70s until now? --It has changed minimally. A long time ago, a teacher had done the entrance test for the student. We have had two instances now of the father doing the test for the student, and one father wanted to come and stay with the student through the program. By and large, while we have students coming from different levels of competitive, before the admittance, we try to catch students whose only interest in math is in winning contests. Often the males come in used to being the best in their classes. We tell students "it is not how much you know, but how much you grow." We don't let them blurt out answers if they have already seen a problem. It puts a burden on us to phrase problems that put students on equal ground. We need to work to keep down students who are too overbearing.

Comment: A good education is to awaken our human sensibility. If what is gives us is disdain for others, we have failed as educators. Elitism is when you foster a group that is arrogant in nature.

Rick Sommer (SUMaC)

I'm here to talk about the Stanford math camp. It was founded in 1995. I've been teaching since it began.

Like the other camps, we're about lots of different things. Part of it is about mathematical enrichment, intensive study, the social environment of having students with similar interests in mathematics all together, living together, eating together, playing together. The average student is in the situation of no one at home understanding them. They are living with undergraduate math majors, math graduate students, math faculty, and there is this common ground. They can talk about a math problem over a game of pool or dinner.

The students spend a couple of hours in class a day. There are two different courses. At the end of class, they get a problem set. They spend a lot more time working on problems than in class, so it is about learning mathematics through doing it. They have study halls in the afternoon. We have a wonderful residence with lots of good common spaces. Student meet with TAs in the afternoon, and student meet their TAs one-on-one a half an hour each day. This allows for a very individualize experience. Even though these are all hotshot math students back home, there is a range of abilities at the program and we can

make adjustments so that every student can be challenged and every student can feel a sense of accomplishment on a daily basis. One day a week, there is a different component, lead by the TAs. It is not original research, but book report research, looking at papers, the internet, or talking to faculty. It is on subtopics of the main course. It culminates in a presentation of the end of the problem. The topics include: [Program I] core material is abstract algebra (organized around motivating problems), constructability, mathematics of the rubix cube, etc. [Program II] algebraic topology. Even though some of the students love to just study pure math, some need more to engage them, and this works for both groups.

We have 35-36 students per year. That is what our residence accommodates. We have about 45% female. We are able to do that because we get many more applications than we are able to accommodate. We get 3 or 4 times as many male applicants. We get 5 or 6 international students and at least a dozen states are represented. We get a lot of immigrants, especially from eastern Europe or Asia. Two years ago students counted that over 20 languages were spoken by the group. There is a geographic diversity and urban/rural diversity. We get a small amount of minority diversity—not as much as we'd like. We're funded by Howard Hughes foundation and offer generous financial aid, but the problem is in recruiting minority students.

We attempt in admission to seek out students who would not have opportunities to do things like this otherwise. If you use AMC results, you overlook students who don't have mathematical opportunities. We try in the recruitment to do what we can to target places where there aren't other opportunities. We used to send letters to every high school in California, but can't any more because of funding. This got students from rural communities that we wouldn't see otherwise.

This benefits the Stanford math department, their people get involved, and it also helps in Stanford recruitment. A lot of their students have their eyes on Stanford for college, but not all of them want to be math majors.

For the students, learning more math is a small part. An appreciation for mathematics and an introduction to life as a mathematician is important. They also develop lifelong friendships. It is routine for them to form an internet discussion group. It is special to me to see how much growth happens during these programs.

I am also deputy director of EPGY. Our primary activity is distance based computer learning. These are courses for students who are ready to go beyond what their schools are offering. You can talk to me to learn more.

Question: How much does it cost to run such a program? The program has been running for 10 years, and every year you have to apply for grants. A lot of sponsors are interested in funding something new and it is hard to get renewal. —Our main funding comes from a Howard Hughes grant through Stanford. The initial funding was for four years, and that has been renewed twice. That provides most of our financial aid. We have other funding from the math department, the AMS epsilon grant, a grant from Ravi Vakil.

Mira Bernstein (Canada/USA Mathcamp)

Mathcamp is a 5 week program with 110 students, we are significantly larger than the other programs we've heard from. We are not affiliated with any university. We change locations from summer to summer. We are hoping to get a three year rotation of locations we like. Mathcamp is run essentially by graduate students. We benefit from advice of senior faculty and have distinguished visitors, but the spirit of the program is the exuberance the grad students bring. Mathcamp originally ran on a small scale, non-residential. As it scaled up, a lot of organization was needed. First, graduate students were counselors who ended up needing to do all the organization.

Originally, students took a test at the beginning which forced them into one of three programs without any choice. The programs were fine, but students we're necessarily in the best one for them. The graduate students who had been organizing things proposed to take over the camp and the proposal was accepted. The present form of Mathcamp was born. The spirit is very much that there is so much math and so many different aspects, and students should have that choice. We bring together a whole bunch of people. The core of the staff is 8 or 9 graduate students and get really excellent applications for these positions. We let them teach whatever they want, however they want, at whatever level. Then one more soul has to put this together into a schedule. Then students are told they can learn whatever they want. We tell them not to try to go to a class in every slot. They choose where they go and when they go and spend the rest of the time doing problems. The classes range in levels and that is marked on the schedule. Every year there is at least one class taught by Moore Method, where students get definitions and theorems and work out the proofs themselves. In the afternoon, there is "cookies and math" which is communal office hour. The instructors though mostly live with the students so this is not the only time to ask questions. Each student has an advisor who consults with them on what they might be doing. Each student gets to decide what they want to get out of the camp. One could get a wide overview and a lot of breadth, or concentrate on one course and learn something really well. You can do contest solving problems, we have two classes on that, and students can do that but we encourage them to also do something else. We bring in non-mathematicians to talk about how math is used in their field and also applied mathematicians.

A couple of words about summer programs in general. The network of math circles is localized into few places. There are vast areas uncovered. The summer programs can serve those kids. If you get a kid from a school that isn't connected to the network, then they make sure that their school gets connected. Mathcamp throughout its history has done one thing always: send a brochure to every high school in the US and Canada. This is extremely expensive and still we know it is not enough. We know that grassroots work is needed to get teachers plugged in but we can only do so much. We get a few students every year from middle of nowhere places and that's great.

All of the programs have financial aid. So send us students, send us all students. Camps cost money, but all the camps give financial aid.

We have four weeks of schedule. For the 5<sup>th</sup> week, we have teachers propose all the material they wanted to teach and didn't get to. Then the students vote on what should be

offered. That really makes sure they come to classes in the 5<sup>th</sup> week, because otherwise they are usually tired by then, but if they voted for the classes then they are more likely to come.

There are problem sets that go with classes and it is up to the professor what they want to do with that. They are not required to do anything ever, but if they do nothing, we would talk to them about that.

Zuming Feng (MOSP)

I'm going to talk about MOP and the difference compared to other mathcamps. I'm distributing problems because that is the most difficult and rewarding part of the program—coming up with problems. MOP is different because it is free to all the students—that is a big draw. That makes us worry very little about our recruitment. After the USAMO, we call the students and students are waiting for that phone call. The camp has been there about 30 years and has been run by different people and it is modified year by year. MOP has a more practical goal—there are immediate results people want to see. Doing well on IMO is not a definite requirement, but we need to keep it in mind. I still believe US education produces most creative students and so we should do well.

MOP has about 30 students on average, but now we have enough support for about 60 students. Last year we had 60. They are coming from different levels. The top 12 USAMO winners are automatically invited because they are competing for the IMO team. The next 18 are the high scorers from 11<sup>th</sup> grade or younger. The next 30 spots are to develop our own future, so we invited the 30 9<sup>th</sup> graders and some very good 10<sup>th</sup> graders to participate.

Instead of telling them they are the top group or whatever, we tell them they are in the black group, blue group, or red group. MOP is 3 to 4 weeks. We work 6 days and have 1 day off. For a regular MOP day, morning classes, afternoon problem sessions and free time, evening lecture. Better to give them free time in the afternoon. During that free time we encourage sports and games.

We have lots of tests during the afternoon--about 12 tests per program. Making the problems for the tests is the hardest part. We have to keep finding the best materials we can find to keep pushing them. We can't do too much in the "real math" area but I hope the problems are intriguing and lead them to that.

The MOP lectures have formats varying among the teachers. Sometimes there are lectures, sometimes students are involved and go to the board, sometimes we talk just about one problem and ten different solutions. I always want the team to be intelligent but also to get away from the eliteness. So I want everyone to feel like they can contribute. We encourage a lot of group work and older students working with younger students. We have lots of team contests. We used Olympiads from around the world and have students work together on teams on the problems.

On the back of the packet: Melanie and Paul Valiant started this. For the ELMO, the older students design a contest for the younger students. The older students organize the

competition, divide into teams, coordinate the solutions. It gives them a chance to write problems and the chance for students to be critical in a friendly way with other students. They get a chance to see how hard it is to give points to things that are badly written and it motivates them to work more on writing their papers up well.

Where do we get our problems? From the monthly, from variations on problems we have seen before, from my own classroom discussions, from ARML and AIME with some enhancements, from our previous students, from math magazines. From problem 14 on, all these problems come from IMO shortlist. All these problems are closely connected to math research right now. This is a trend, good problems coming from research. This is our greatest need—from the research community we need more problems that come up in their research. We need good problems to motivate our kids and get them interested in mathematics.

The AMC needs problems of all levels. We need more problems to bring to the IMO committee also.

Abigail Thompson (COSMOS)

COSMOS is a four week summer program in math and science. There are four campuses at the U of California that run COSMOS programs. Each site has 150 students. The selection criterion is academic excellence. This is extremely unpopular criterion with the university. This program is different by being in math and the science. The students are in groups of roughly 20 and each group is in a particular area: geology, math, etc. A lot of the regular research faculty from the campuses teach in the program. The secret to getting them is easy: pay them a lot. I don't think we'd get them any other way. The challenge is to get them to do something other than lecture.

The program is \$1600, a bargain for California residents. 1/3 on full or partial scholarship based on need. We have high school math or science teachers work on a course on writing. Each student produces a project over the course of the month. The high school teachers who work for the program are also well paid. Last year at one camp we had 400 applications for 150 spots. We are getting the students we'd like to be getting. The goal of the program is to salvage the few. We are not producing enough mathematicians or scientists. Fixing this is a huge problem, but we are trying to do something by reaching students who have potential to go onto PhDs in math or sciences. We have to make a special effort to reach students who would not maybe go into math or science otherwise.

Slightly more girls than boys overall, though it varies by subject. The girls are overwhelmingly interested in biology and not in engineering. Math is about even. They are all interested in biology to help people or take care of people.

We are able to support students who do not have continuous support from their parents. In many of these other programs, their parents are supporting them—and that is a limited group of students. That explains a lot of the demographic issues that have come up.

I tell the parents that students who are good in math and science feel isolated and this can provide community for them. A drawback is this intense sense of community—which

can be distracting. Another drawback is unstable students, other issues that come up with having 150 high school students together for a month.

We need to make more of an effort to convince research mathematicians that this is important. For my program it is hardest to get math faculty to teach. In other fields, it is considered more standard. In math, it is considered a foolish thing to do.

A couple of phrases that make me anxious are “exceptional mathematical talent” and “profoundly gifted.” Unless we are only looking for the young Bill Thurston, these are exaggerations. We should be looking for kids who are very good at math, and are really really very interested in it, and love it, and want to spend time on it.

Question: What does it cost per week to run your program?

Mathcamp: \$600/per student per week,

Hampshire: \$600/per student per week

Ross/ ProMYS: probably on par (a 6 week program cost \$250,000)

SUMAC: \$800 per student per week

COSMOS: a lot more because biology is expensive

What do you look for in students to accept to your program?

COSMOS: grades in math and science, taking advanced math and science classes, recommendations

SUMAC: test scores, grades in school, recommendation, admissions exam (math problems), questions about why they want to come, we look for students with their heart in the right place; we try to get a geographically and culturally diverse population

Ross/ProMYS: writing up solutions to problems with open endedness, from which you can tell a lot about their background

Hampshire: recommendation, letter of interest from student, a test that introduces student to some new idea with some deeper questions about finding a pattern—everything is about “Try this” or “See this”—we don’t ask to “prove” because that turns off many students. We ask the students to comment on the test itself and that is very revealing. A student who find it “unfair” is unlikely to get in. If there is anything there that makes me want to pursue the student further, I give them another interesting test, and am open to further discussion. We get 4-5 applications per spot, about half of those don’t submit the test.

Mathcamp: we don’t look at anything from school, we have a qualifying quiz, we try to make the problems look interesting to draw students in, we say “explain how you got your answer, we want to see conjectures and partial results”, letters of recommendation, letter of interest from student, they can get in with a low score on quiz but a lot of enthusiasm