PERCOLATION OF BINARY SEQUENCES

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I. Benjamini, H. Kesten, **AP**, Vol 23, (1995)

Graph $\mathcal{G}$: $\mathbb{Z}^d$, $\mathcal{T}$, $\mathbb{Z}^2_{cp}$

$\mathcal{V}$ - vertex set of $\mathcal{G}$,

P_p the product measure on $\Omega = \{0,1\}^{\mathcal{V}}$

$$P_p(\omega_v = 1) = p$$

$$\Xi = \{0,1\}^{\mathbb{N}}$$

A generic element $\xi = \{\xi_1, \xi_2, \dots\}$ is called *word*.

Definition. Word ξ is *seen from the vertex v in the configuration ω* if there exists self-avoiding path $(v, v_1, v_2, \dots)$ on $\mathcal{G}$, starting from v, such that

$$\omega_{v_i} = \xi_i, \; i \geq 1.$$

$$S(v) = S(v, \omega) = \text{collection of words which are seen from } v \text{ in configuration } \omega$$

$$S_\infty = \cup_{v \in \mathcal{V}} S(v, \omega).$$

Theorem. (Benjamini, Kesten) Let $\mathcal{G} = \mathbb{Z}_+^d$, and $p = 1/2$. Then for $d \geq 10$,

$$P\{S_\infty = \Xi\} = 1,$$

and for $d \geq 40$

$$P\{S(v) = \Xi \text{ for some } v\} = 1.$$

μ - the product measure on Ξ: $\mu\{\xi = 1\} = \mu$.

For each $\xi \in \Xi$

$$\rho(\xi) = P\{\xi \text{ is seen from some } v\} = 0 \text{ or } 1 \ .$$

and

$$\mu\{\xi : \ \rho(\xi) = 1\} = 0 \text{ or } 1 \ .$$

(in case it is 1, we say almost all words percolate)

Theorem. (Kesten, Zhang, S.) Almost all words are seen on triangular lattice at $p = 1/2$.

Theorem. (Kesten, Zhang, S.) All words are seen on $\mathbb{Z}^2_{cp}$ lattice for $p_c(\mathbb{Z}^2_{cp}) < p < 1 - p_c(\mathbb{Z}^2_{cp})$.

Open problems: $p = 1/2$ and $\mathcal{G}$ is $\mathbb{Z}^2$, $\mathbb{Z}^3$.

What happens if p is close to 0 or 1 (is sub- or super-critical)?

Theorem. (Kesten, Vares, S.) Almost all words are seen on $\mathbb{Z}_+^2$ for $p > p_c(\mathbb{Z}_+^2, oriented)$, if μ is sufficiently close to 1.

We consider the following north-east oriented site percolation model on $\mathbb{Z}^2_+$: the lines

$$H_i := \{(x,y)\colon x+y=i\}$$

are first declared to be "bad" (or "good"), independently of each other, with probability δ ($1-\delta$, respectively);

vertices on good lines are open with probability

$$p_G > p_c(\mathbb{Z}^2 \textit{ oriented}),$$

vertices on bad lines they are open with probability

$$p_B < p_c(\mathbb{Z}^2 \textit{ oriented}),$$

independently of each other, given the configuration of lines.

$$\Theta(p_G, p_B, \delta) \;=\; P(C_0 \text{ is infinite }),$$

where C_0 denotes the oriented open cluster of the origin.

Theorem(Kesten, Vares, S.) If $p_G > p_c$ and $p_B > 0$ there exists $\delta_0 = \delta_0(p_G, p_B) > 0$ so that

$$\Theta(p_G, p_B, \delta) > 0$$

for all $\delta \leq \delta_0$.

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