w/ A. Chang & P. Yang

$n=4 \quad (M, g)$

$R_c$ : Ricci

$R_m = R_{ijkl}$

$$(*) \begin{cases} \Delta R_{ij} = (R_c * R_m)_{ij} & \text{in } M \\ \nabla_n R_{\alpha\beta} = 0 & \text{on } \partial M \\ L_{\alpha\beta} = 0 & \text{on } \partial M \Rightarrow R_{\alpha n} = 0 \end{cases}$$

$\uparrow$ II FF $\qquad$ Dirichlet + Neumann

$$\Delta R_{ijkl} = (\nabla^2 R_c)_{ijkl} + (R_m * R_m)_{ijkl}$$

Thm A (CCY)

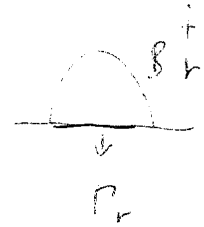
$$\exists\, \varepsilon > 0, \quad \int_{B_r^+} |R_m|^2 \le \varepsilon^2 \Rightarrow \sup_{B_{r/2}^+} |\nabla^k R_m| \le \frac{C_k \varepsilon}{r^{2+k}}$$

- without bdy Tian-Viaclovsky ('05)

$$(**) \begin{cases} \Delta R_c = R_c * R_m & M \\ R = \text{const} & M \\ \nabla_n R_{\alpha\beta} = F_{\alpha\beta} & \partial M \\ L_{\alpha\beta} = 0 & \partial M \\ W_{\alpha n \beta n} = 0 & \partial M \end{cases}$$

~~R const~~

Thm (B) (CCY) $\exists\, \varepsilon$, $\int_{B_r^+} |R_m|^2 \le \varepsilon^2$, $\oint_{\Gamma_r} |F| \le \varepsilon$




$$\| \hat{\nabla}^i \hat{R}_m \|_{L^\infty(\Gamma_r)} \le \varepsilon, \quad i = 0, 1, 2, \Rightarrow \sup_{B_{r/2}^+} |R_m| \le \frac{c\,\varepsilon}{r^2}$$

$\hat{R}_m = R_{\alpha\beta\gamma\delta}$, $\hat{\nabla}$ tangential derivative

Motivation: Einstein, Kähler, · General relativity, $F_{\alpha\beta} \sim$ mass

Q: find matching bdy condition for a system ?

$\rightsquigarrow$ - - - - - - - - - - - a fully nonlinear eq ?

Yamabe = $\hat{g} = V^{\frac{4}{n-2}} g$, $R_g = \text{const}$ $h_g = 0$

$$\begin{cases} \Delta V - C_n R_g V + \lambda C_n V^{\frac{n+2}{n-2}} = 0 & n \geq 3 \\ \frac{\partial V}{\partial n} = C_n' h_g V \end{cases}$$

$n = 2$

$4\pi\chi = \int K + \oint k_g$

· Escobar ('92)

Nonlinear $A = \frac{1}{n-2}\left(R_c - \frac{R}{2(n-1)} g\right)$

$\sigma_1(A) = \operatorname{tr} A = \frac{1}{2(n-1)} R$

$$\begin{cases} \sigma_k(\nabla^2 u + \text{gradien} + \cdots) = e^{2ku} \\ ? \end{cases}$$

$\beta = \frac{1}{2}\beta^2 + \mathcal{L}$

↓ mixed system ↑ l.c.i

Q: ( $\sigma_k(A)$, ? )

$n = 4$ $32\pi^2 \chi = \int \underbrace{|W|^2}_{\text{locally conf inv}} + 16\left(\underbrace{\int \sigma_2(A)}_{\text{system}} + \oint_{\partial M} B\right)$

<u>Thm</u> · (C. thesis) $n \neq 4$ $g \xrightarrow{\text{conf}} \int \sigma_2 + \oint B^2$ with vol = 1

$$\Leftrightarrow \begin{cases} \sigma_2 = \text{const} & M \\ \beta^2 = 0 & \partial M \end{cases}$$

Ans. $(\sigma_2, \beta^2)$ AND $(|W|^2, \mathcal{L})$

without bdy $g \mapsto \int |W|^2 \iff B_{ij}=0$ (Bach 21)

$$B_{ij} = \Delta R_{ij} - \frac{1}{6}\Delta R\, g_{ij} - \frac{1}{3}\nabla^2_{ij} R + (R_c * R_m)_{ij}$$

$B_{ij}=0$ Not Elliptic

$R=\text{const} \Rightarrow B = \Delta R_c + R_c * R_m$ (conf inv tensor)

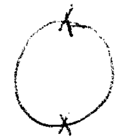
$g \mapsto \int |W|^2 + 16 \oint \mathcal{L}$ first variation

Def: $S_{\alpha\beta} = \nabla^i W_{i\alpha n\beta} + \nabla^i W_{i\beta n\alpha} - \nabla^i W_{n\alpha n\beta} + \frac{4}{3} h\, W_{\alpha n\beta n}$

conf inv tensor on $\partial M$

Let $R=\text{const}$, $L_{\alpha\beta}=0 \Rightarrow S_{\alpha\beta} = -\frac{1}{2} \nabla_n R_{\alpha\beta}$

$\oint_{\partial M} |S|$ conf inv — $L^1$ norm of F

Note: 3-d space Schuarzchild $\overset{\text{conf}}{\rightsquigarrow}$ 

4-d Ad, Scharzchild $\overset{\text{conf}}{\sim}$ compact mfld with bdy sat (* *)

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Prove thm A

<u>Thm C</u> (CCY) $(\nabla^* R_m)^{odd}=0$, $(\nabla^* R_c)^{odd}=0$ on $\partial M$

where $\nabla_{i_1}, \nabla_{i_k}, R_{i_{k+1}, i_{k+4}} \in (\nabla R_m)^{odd}$ if

$(i_1,\cdots,i_{k+4})$ has odd # of n

C ⇒ A $\oint \underbrace{\nabla_n R_{ij} R_{ij}}_{\text{odd}}$, $\oint \underbrace{\nabla_n R_{ijkl} R_{ijkl}}_{\text{odd}}$ ...

pf of C

Parity

| |
|---|
| odd + odd = even |
| odd + even = odd |

$k = 0, 1$ true

$\nabla_\alpha (\nabla R_c)$ o.k

$\nabla_n (\nabla_\alpha R_c) \sim \nabla_\alpha (\nabla_n R_c)$ o.k

$\nabla_n \nabla_n R_c$

$$\Delta R_c = R_c * R_m$$

$$\nabla_n \nabla_n R_c + \sum_\alpha \nabla_\alpha \nabla_\alpha R_c$$

$$\Delta R_m = \nabla^2 R_c + R_m * R_m$$

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