1/24/08 Michel Broué # 2

~~add notes later~~ .. (these are notes to augment the prepared slides)

Start w/ review of last time ...

$$|GL_n(q)| = q^{\frac{n(n-1)}{2}} \prod_{d \le m} \phi_d(q)^{[\frac{n}{d}]} \leftarrow = m \qquad n = md + r, \quad r < d.$$

Choose d

$|T_c| = q^d - 1$

$$\underbrace{GL_d(q) \times GL_d(q) \times \dots \times GL_d(q)}_{m \text{ times}} \times GL_r(q)$$

coxeter torus in each $GL_d(q)$ $\longrightarrow$

$$T_c \times T_c \times \dots \times T_c$$

$$S_d = C_{\phi_d(q)} \times \dots \dots \times C_{\phi_d(q)}$$ this is our Sylow subgroup (ies)

take l prime

suppose $l \mid |GL_n(q)|$ and that $l > n$ (so $l \nmid |W|$

moreover suppose $l \nmid q$.

then

$\exists!\ d$ s.t. $l \mid \phi_d(q)$. (easy ex. on cyclotomic polys ϕ_d)

the l-sylow of $GL_n(q)$ is the l sylow of S_d $\leftarrow$ the ϕ_d-sylow

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Compare normalizer $\to N_{GL_n(q)}(S_l)$ $\leftarrow$ l-sylow $\underline{\underline{\text{to}}}$ $N_{GL_n(q)}(S_d)$

and

centralizer $\to C_{GL_n(q)}(S_l)$ $\underline{\underline{\text{to}}}$ $C_{GL_n(q)}(S_d)$

it turns out we get equalities

But RHS is a centralizer of a torus, so its a Levi
(in fact its a minimal d-split ~~torus~~ levi)

~~so~~ we have cyclic group

$\phi_d(q) \times \dots \times \phi_d(q)$ sitting in the torus $(q^d-1) \times \dots \times (q^d-$

and that sits in $GL_d(q) \times \dots \times GL_d(q) \times GL_n(q)$

So $N/C = C_d \wr S_n \leftarrow ?\ \ G_n?$ (wreath)

cyclic of order d

Consider $d=1$ case then $\phi_1(q) = q-1$.

the $q-1$ sylow $S_1 = \begin{pmatrix} * & & 0 \\ & \ddots & \\ 0 & & * \end{pmatrix}$ "split torus" $|S_1| = (q-1)^n$

so here

$$N_{GL_n(q)}(S_1) \Big/ C_{GL_n(q)}(S_1) = \underset{\text{weyl}}{W} = \underset{\text{symmetric group}}{S(n)}$$

$$L = \underbrace{GL_1(q^d) \times \dots \times GL_1(q^d)}_{m\text{-times}} \times GL_r(q)$$

$N_G(L)/L$ is a finite group ... can see from RHS.

call the "relative Weyl group"

$G(q)$ w/ subgroup $L(q)$ then

Claim: $G(q)/L(q)$ is a polynomial (Cauchy Theorem)

So look at $\left.\dfrac{G(q)}{L(q)}\right|_{q=\xi_d} = |W_G(L)|$

primitive dth root of unity

when $d=1$ get $n!$ (in case where $G = GL_n$)

"" S_n" is GL_n over field w/ 1 element ..."

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compute $|G|/|L| = |G| \Big/ |C_G(S)|$ = $|G| \Big/ |N_G(S)| \cdot |N_G(S)| \Big/ |C_G|$

centralizer of a sylow

$= |G| \Big/ |N_G(S)| \cdot \left| N_G(S) \Big/ C_G(S) \right|$ ← order of a fin. group.

So $|G| \Big/ |L| \equiv |W_G(L)| \mod \phi_d$

~~Also~~ So its not by chance that we found Weyl group (Springer-Lehrer)

they occur as ϕ_1-cyclotomic weyl groups in split case

Unipotent Characters

for every finite $G(q)$ there ~~are~~ unipotent characters

$\mathcal{U}n(G(q)) \subseteq Irr(G(q))$ irr. chars.

(normally, unipotent characters are parametrized by partitions)

each generic uni character comes w/ a poly. degree

Recall $GL_n(-q) = $ "$U_n(q)$" up to $\pm$

if instead you evaluate ~~Deg~~ ... at $x=-q$ you get "degree of unip. character

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in example GL_n

$Deg_\chi(x)$ <u>is</u> actually a poly.

For any finite reductive group, say $\mathcal{U}n(G)$

and $\gamma \in \mathcal{U}n(G) \rightsquigarrow Deg_\gamma(x) \in \mathbb{Q}[x]$ in general

look at $x=q$ $G(q)$ $\gamma_q \rightsquigarrow \gamma_q(1)$ but in GL_n the coefs. are in $\mathbb{Z}$

Deligne-Lusztig induction & restriction

Functor sends graded module to graded module...

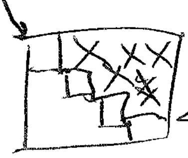
Let L be a 1-split Levi i.e. a centralizer of a ϕ_1-subg

e.g. in GL_n

for d-split get q^d

$GL_{m_1}(q) \times GL_{m_2}(q) \times \cdots \times GL_{m_s}(q)$ where $m_1 + \cdots + m_s = n$

is a parabolic subgroup in GL_n



$= P = \mathcal{U} \rtimes L$ (unipotent)

So For L 1-split (i.e. $\exists$ P s.t. $P = \mathcal{U} \rtimes L$) $P \twoheadrightarrow L$

then

R_L^G is easy to compute, take Y and L-module view it as a P-mod via $P \twoheadrightarrow L$ and then

induce Ind_P^G

in this case it is known as Harish-Chandra induction

consider type $\mathbb{G}$ and subgroup type $\mathbb{L}$
choose q and look at $G(q)$, $L(q)$
perform DL induction $R_{L(q)}^{G(q)}$ look at $\overline{G(q)}$ abo
$\overline{L(q)}$

$\overline{L(q)}$ has $\overline{U}$ and $\overline{P}$.

not unique so choose 1

gives a variety $X(\overline{U})$

var w/ pairs

$G(q)$ $L(q)$

choose l compute

$H_c^{x}(X(\overline{U}))$

w/ compact support

its a graded bimodule for

$G(q)$ $L(q)$

you get same result regardless of choice of q, L, or U.