recall we're discussing $\Lambda^k = \text{alg. of poly's on partitions}$

it has 3 different bases

$S_k^*$ - interpolating schur functions

$P_k^*$ - central characters

by Fourier trans $\Lambda^k \cong \bigoplus_{k=0}^{\infty} \mathbb{Q}[S(k)]$

recall $p_k^* (\lambda) = \sum_{i} \left[ (\lambda_i - i + \frac{1}{2})^k - (-i + \frac{1}{2})^k \right]$

today we want to understand how to write $F_k$'s in terms of $P_k^*$

we'll try to create a fictitious element in $\bigoplus_{i \geq 0} \mathbb{Q}$

s.t. $P_k^* (\lambda)$ is the central character

of the completed k-cycle $\frac{1}{k} \mathbb{C}[k] + \text{smaller permutations}$

---

Gromov - Witten --- theory

Start w/ var. $\overline{M}_g (X, d)$ smooth proj.

we want to understand alg. curves inside

we have $\overline{M}_g (X, d) \to F_k (\mathcal{C}_1) = \alpha \in H^2(X)$

a two-cycle

Grassmannians instead think of vector space $\mathbb{C}^n$

instead $[\mathbb{C}^n, \mathbb{C}^k] \to \mathbb{C}^k$

a linear map $\mathbb{C}^n \to \mathbb{C}^k$

So the goal of G-W--- theory is to look at

Schubert calculus in the setting above...

i.e. define cohomology class and compute intersections

For today dim $X = 1$ i.e. $X$ is a curve

given smooth curve $\overline{\mathcal{C}_1} \to \mathcal{C}_1$ and map $\xi$ (a branched covering)

the topology uniquely defines an algebra

(unique structure upstairs makes this map belong)
locally over a pt. to every pt. \(q \in X\) we associate a partition of \(d\) (a called profile or monodromy (each component in pre-image is a part)

Hurwitz problem: enumerate all branched covers with the given monodromy of cycle type \(\mu^{(1)}, \ldots, \mu^{(n)}\) by set

\[
\sum_{\text{branched covers } f} \frac{1}{\text{Aut } f_1} = \# \text{ homomorphism } \pi_1(X \setminus f_2) \to \text{S}(d)
\]

(Branch cover means an unramified cover when the branch pts.)

\[
h = \text{genus of } X
\]

So if \(h = 0\) then we get \((d!)^2 \cdot \text{tr} \in \text{S}(d)

for \(h > 0 \) \[
2 - 2h = \sum_{\text{char}} (\dim \chi)^2 \cdot \text{dim } C \chi
\]

for \(h < 0\) so how do we make Alg.Geom. out of Hurwitz prob??

now we need to look at equations...

Var. \(Y\), write down equation... think about \(x^3 + y^3 + z^3 = 0\) its a section of \(\Theta(3)\) (eqns are sections of sheaves)

How do we write an equation for "ramified over \(g\)?

\[
\text{want } f \text{ to be ramified over } g
\]

clearly, \(f(p) = g\) \(\Rightarrow f'(p) = 0\).
So $f'(p) \in T^*_p C$ is a cotangent line at $C$ at $p$.

(i.e. \( \forall p, T^*_p C \) is a line ... ?)

$T^*_p C$ is a line bundle over $\overline{M}_{g,1}(X, d)$ call it $\mathcal{L}$

(\( \mathcal{L} \) stands for \( \text{unmarked pt}$, e.g. \( p \))

if we pick a section the locus where it vanishes

minus locus where it blows up is called a Chern-class

So \( \delta f'(p) = 0 \) is \( C_1(\mathcal{L}) \) on the smooth locus

(an elliptic curve such as $x^3 + y^3 + z^3 = 0$ is a rep. of the 1st Chern class)

Now, how do we say $f$ has triple ramification?

$F'(p) = 0$ and $F''(p) = 0$. (second deriv)

section of $\mathcal{L}$ section of $\mathcal{L} \otimes \mathcal{L}$ but this is

so let's take $C_1(\mathcal{L}) \cup C_1(\mathcal{L}^2) \sim 2C_1(\mathcal{L})$

so $= 2C_1^2$.

How about 5-triple ramification? \( \delta \) ! $C_1$

The (k)-cycle ram over $\mathcal{L}_i$ Condition of the smooth part of $\overline{M}_{g,n}(X, d)$ is a representative of the

\( (k-1)! \cdot C_1(\mathcal{L}_i)^{k-1} \). \( \mathcal{L}_i = T^*_p C \)

On the whole of $\overline{M}_{g,n}(X, d)$

\( C_1(\mathcal{L})^k = \frac{1}{(k+1)!} \) completed (k+1)-cycle.
Theorem (Okounkov-Pandharipande)

\[ \int \prod C_1 (\mathfrak{m}_i)^{k_i} = \sum \left( \frac{\dim \lambda}{d!} \right)^{2-2g_{\text{can}}(X)} \frac{\prod P_{k_i+1}^*(\lambda)}{(k_i+1)!} \]

\[ \text{virt.} \quad \frac{[\text{M}_g(X,d)]_{\text{virt.}}}{\text{virtually...?}} \]

From the modern perspective this is one of the simplest instances of the GW/DT correspondence.

(formulas like the one above appear in the geometry of the Hilbert scheme of pts. of \( \mathbb{C}^2 \).)

The correspondence has to do with the case when \( \dim X = 3 \).

i.e. consider \( X = \mathbb{C}^2 \)

\[ \text{study curves that project to curves in } \mathbb{C}^2 \text{ and pts. in } \mathbb{C}^2. \]