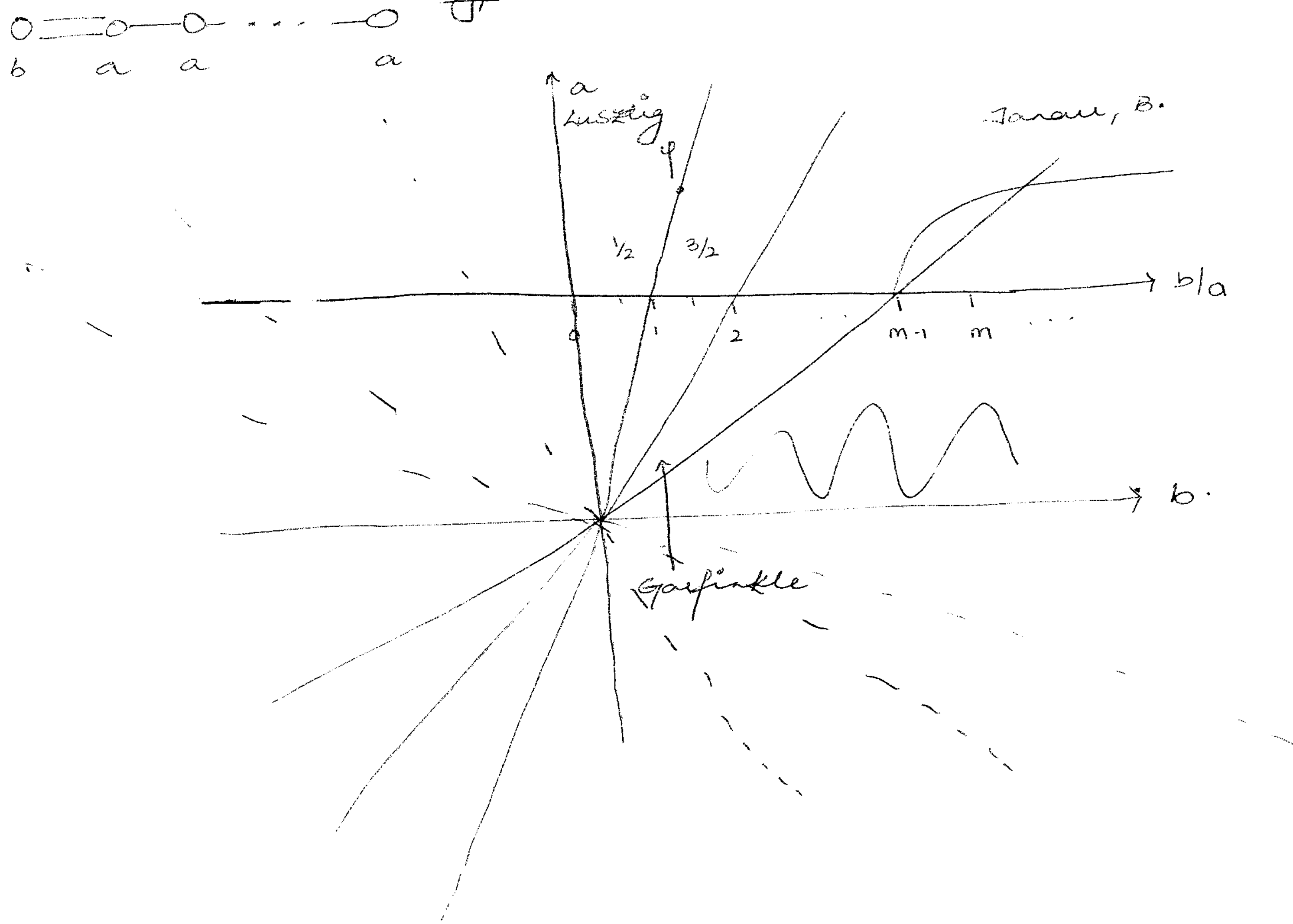
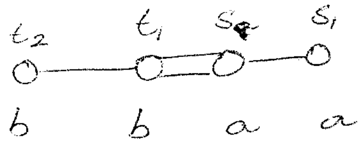
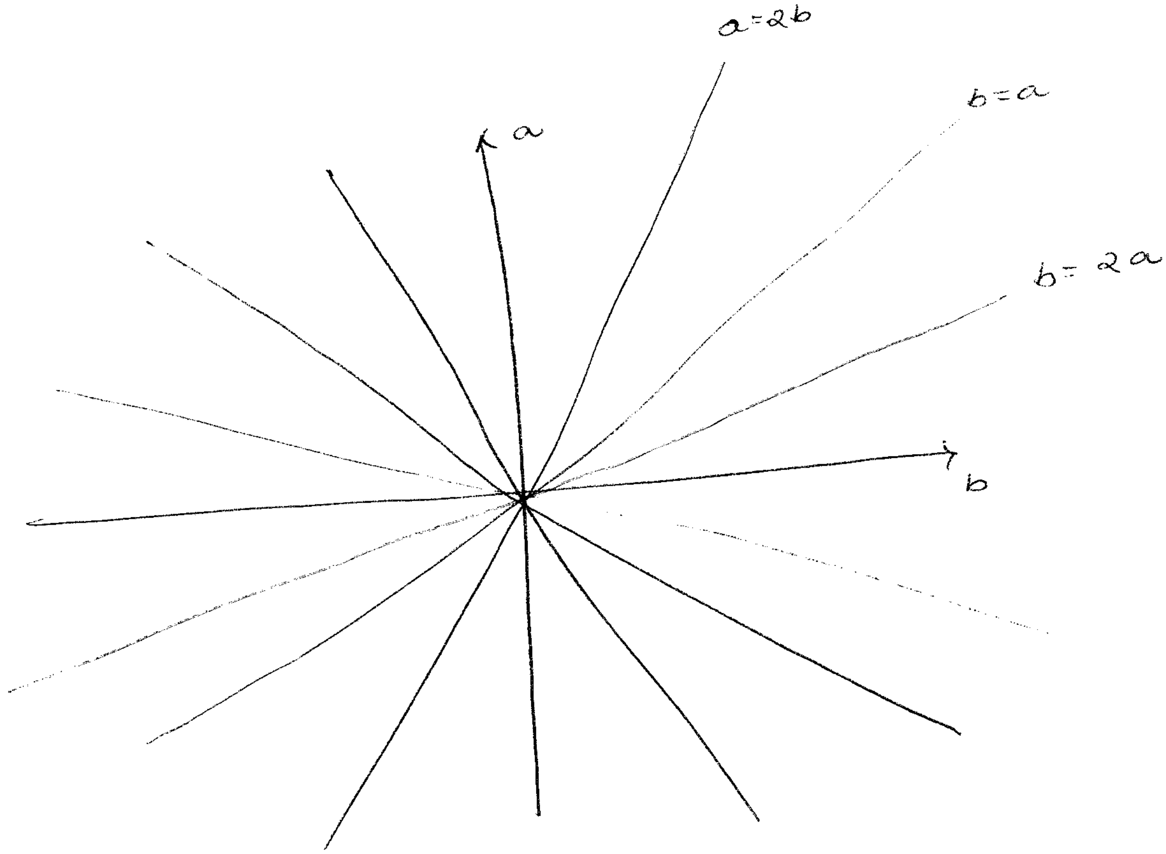
Type B



Type F_4



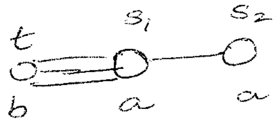


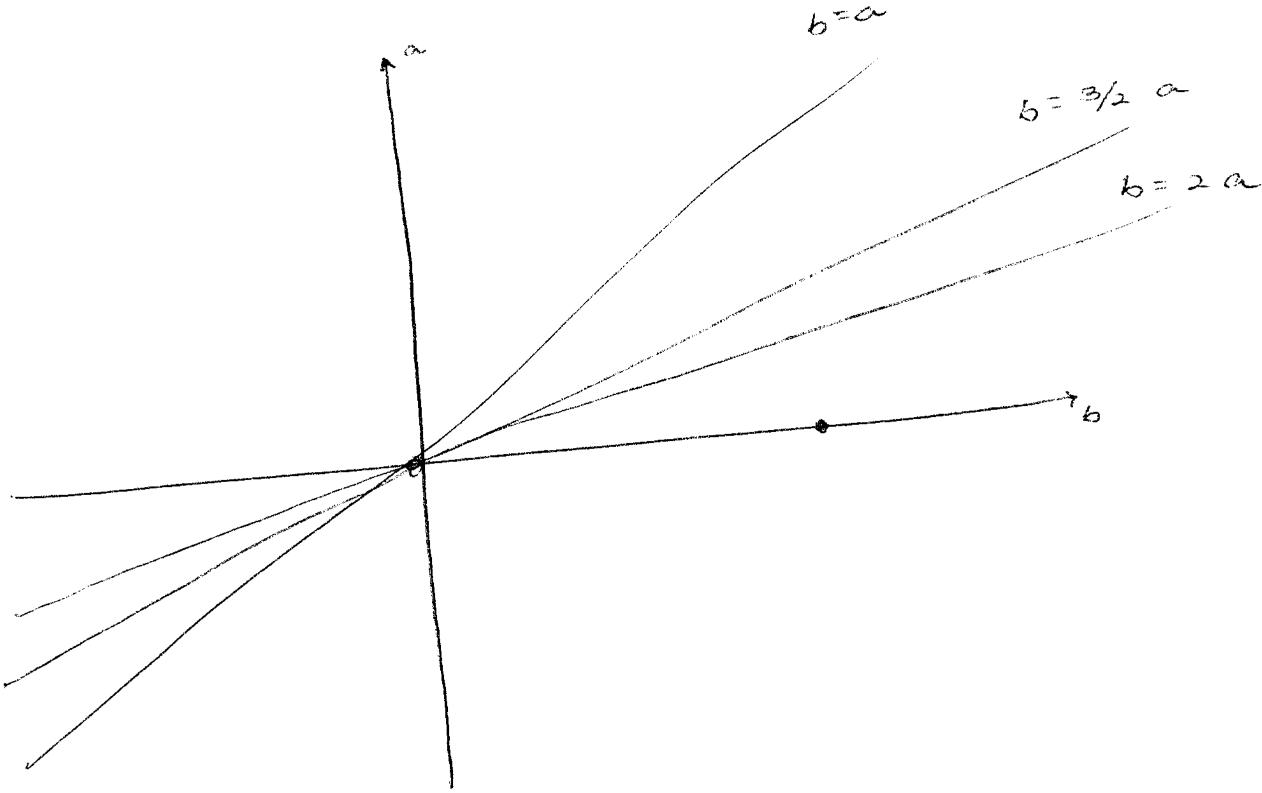
$W = W_\varphi \ltimes \tilde{W}$, $\tilde{W} = \langle \tilde{T} \rangle$

$\mathcal{C} = W_\varphi \cdot \tilde{\mathcal{C}}$, $\tilde{\mathcal{C}}$ left cell in $\tilde{W}$

$W_\varphi \cdot \{1\} \leftsquigarrow \{1\}$ is a left cell

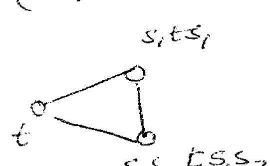
Type $\tilde{G}_2$





$Ba = 0$, $\tilde{W}_\varphi = S_3$

$W(\tilde{G}_2) = S_3 \ltimes W(\tilde{A}_2)$



Semicontinuity properties of Kazhdan-Lusztig cells

Cédric Bonnafé

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MSRI (Berkeley) - March 2008

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- **Involution:** $\overline{v^{\gamma}} = v^{-\gamma}$, $\overline{T}_w = T_{w^{-1}}^{-1}$

Theorem (Kazhdan-Lusztig 1979, Lusztig 1983)

If $w \in W$, there exists a unique $C_w \in \mathcal{H}$ such that

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Corollary

Since $C_s = T_s$ and $C_{sw} = C_s C_w$ for all $s \in S_\varphi$ and $w \in W$, the left cells of (W, S, φ) are of the form $W_\varphi \cdot \mathcal{C}$, where $\mathcal{C}$ is a left cell of $(\tilde{W}, \tilde{I}, \tilde{\varphi})$.

Examples

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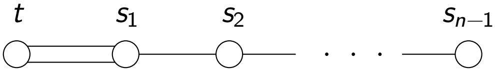

$$W(B_n) = \mathfrak{S}_n \times (\mathbb{Z}/2\mathbb{Z})^n$$

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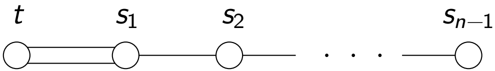

$$W(B_n) = \mathfrak{S}_n \times (\mathbb{Z}/2\mathbb{Z})^n = \langle t \rangle \ltimes W(D_n)$$

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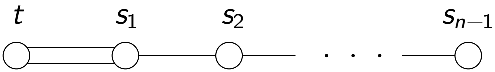

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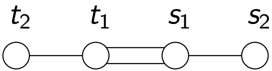
- Type F_4



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Examples

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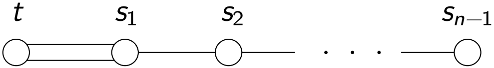

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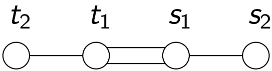
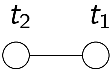


$=$



$\ltimes$

$$W(F_4) = \mathfrak{S}_3 \ltimes$$

Examples

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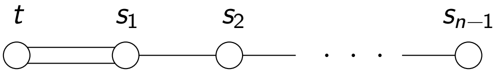

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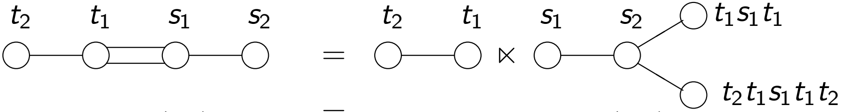

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Type B

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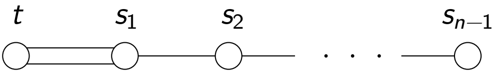
$(W, S) = (W_n, S_n)$, where $S_n = \{t, s_1, s_2, \ldots, s_{n-1}\}$ and



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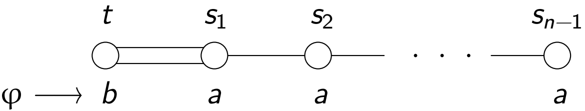

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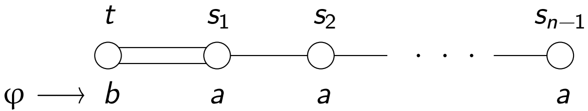

We identify W_n with the group of permutations w of $I_n = \{\pm 1, \pm 2, \ldots, \pm n\}$ such that $w(-i) = -w(i)$ through

$$t \mapsto (-1, 1) \qquad \text{and} \qquad s_i \mapsto (i, i+1)(-i, -i-1)$$

Domino insertion algorithm:

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Example: $w = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & -8 & -9 & 1 & 6 & -4 & 5 & 3 & -2 \end{pmatrix} \in W_9$

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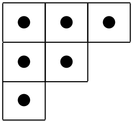

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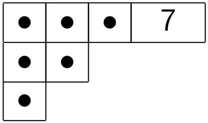

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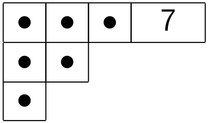

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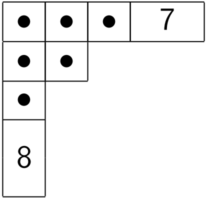

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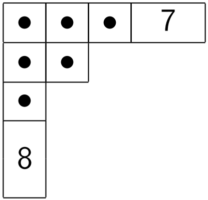

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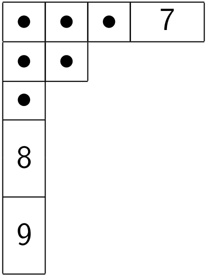

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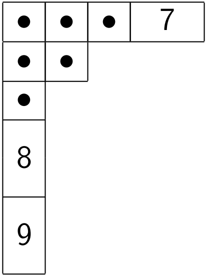

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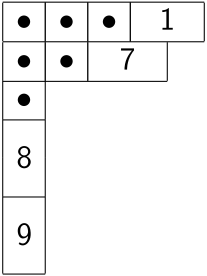

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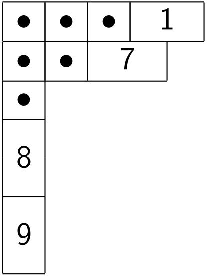

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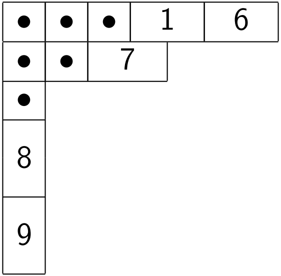

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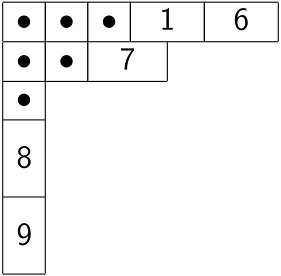

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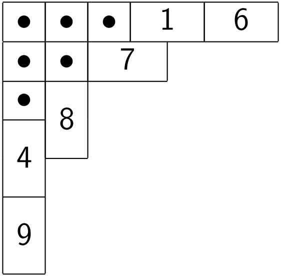

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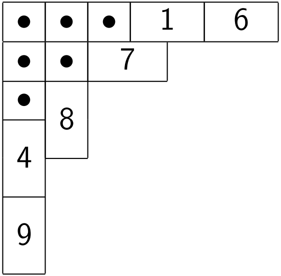

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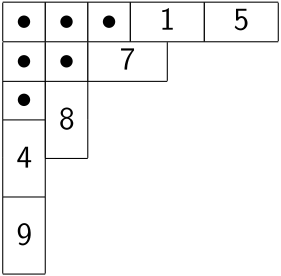

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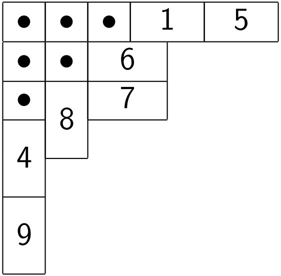

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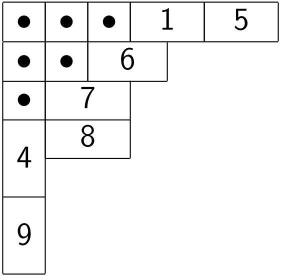

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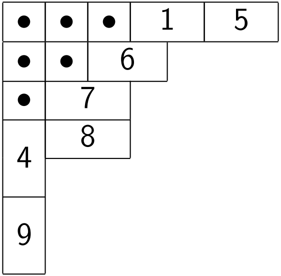

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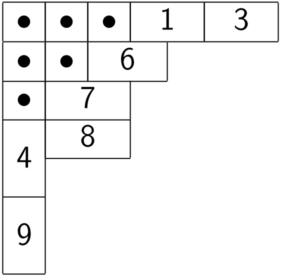

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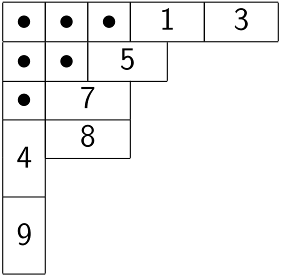

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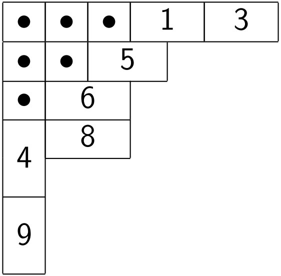

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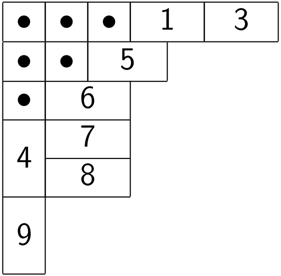

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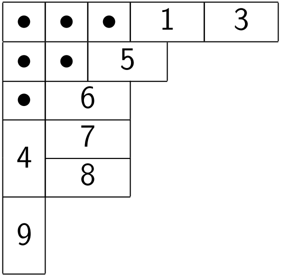

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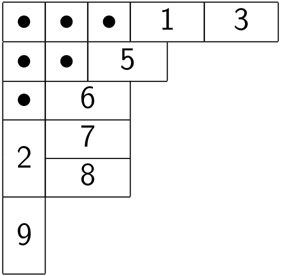

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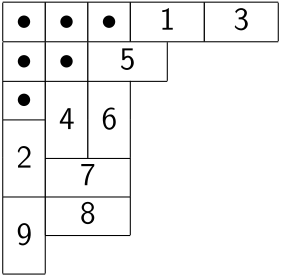

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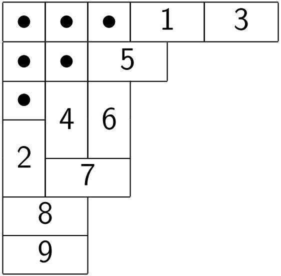

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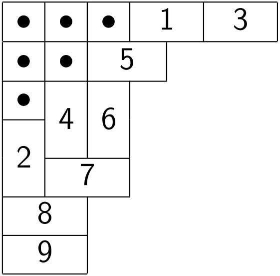

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Theorem (?)

$$\begin{array}{rcl} W_n & \xrightarrow{\sim} & SDT_r^{(2)}(n) \\ w & \longmapsto & (D_r(w), D_r(w^{-1})) \end{array}$$

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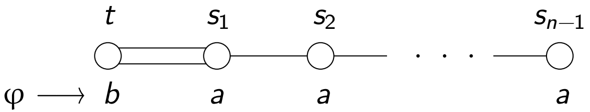

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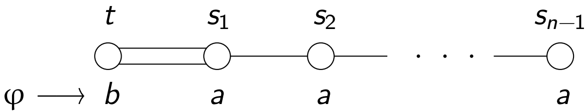

Conjecture A (Geck-Iancu-Lam-B. 2003)

Assume a, $b > 0$ and assume that $0 \leqslant r < b/a < r+1$. Then:

- $w \sim_L w'$ if and only if $D_r(w^{-1}) = D_r(w'^{-1})$
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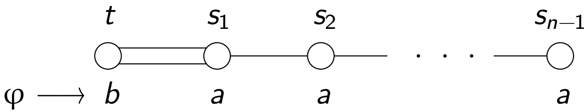

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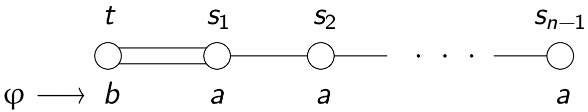

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Conjecture C (maybe only for finite or affine Weyl groups)

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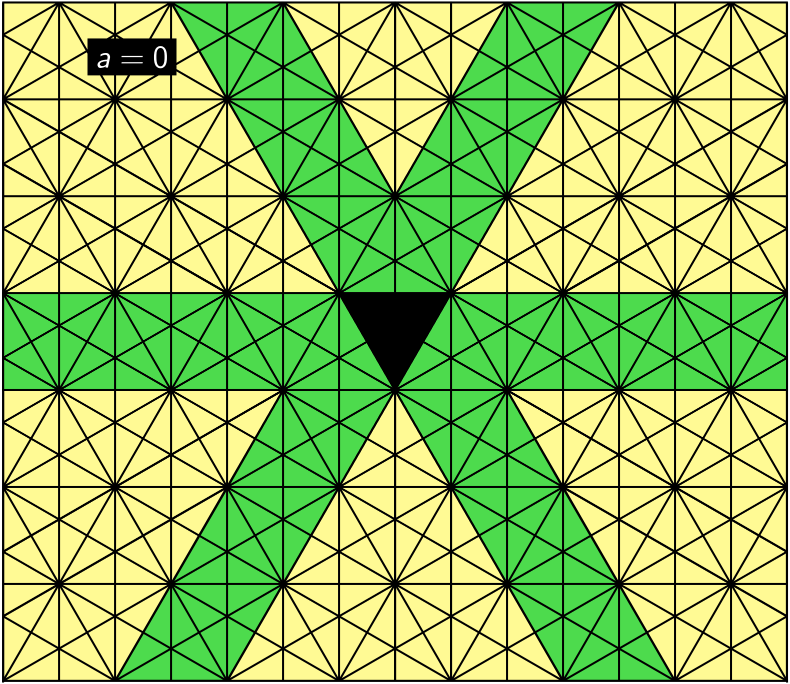
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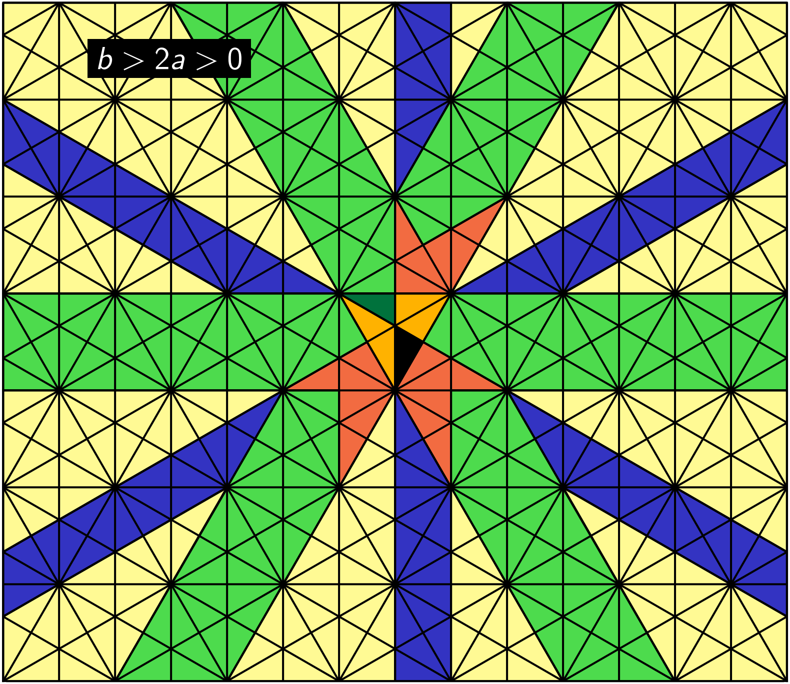
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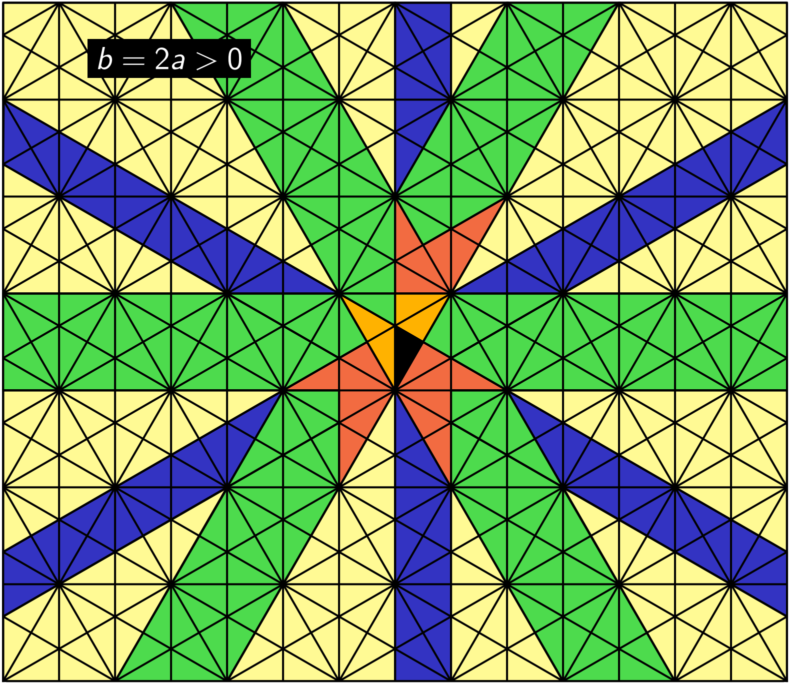
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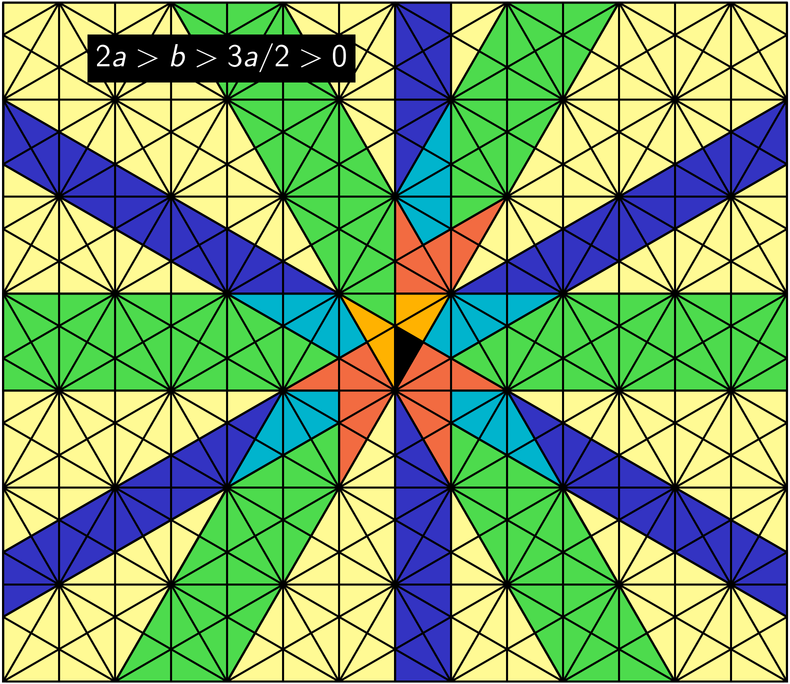
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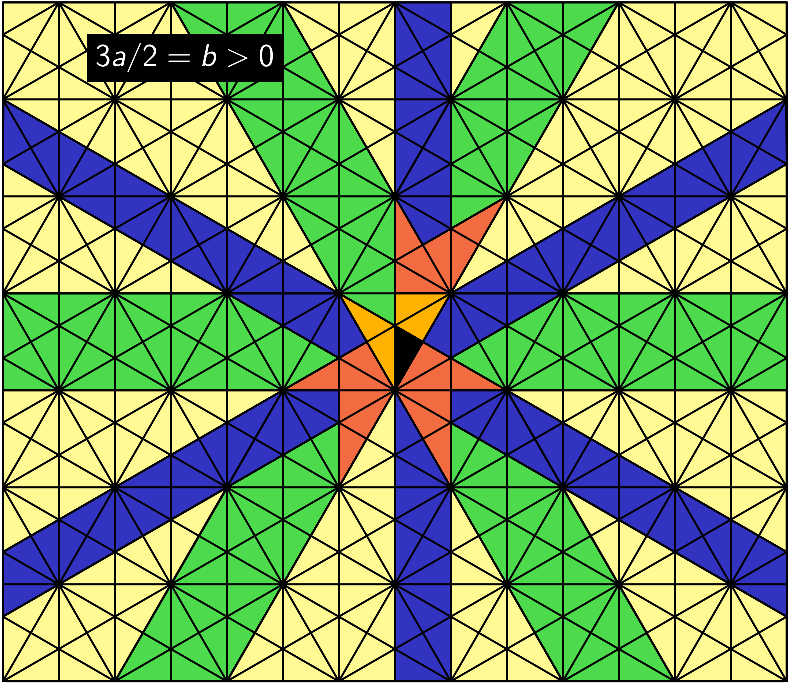
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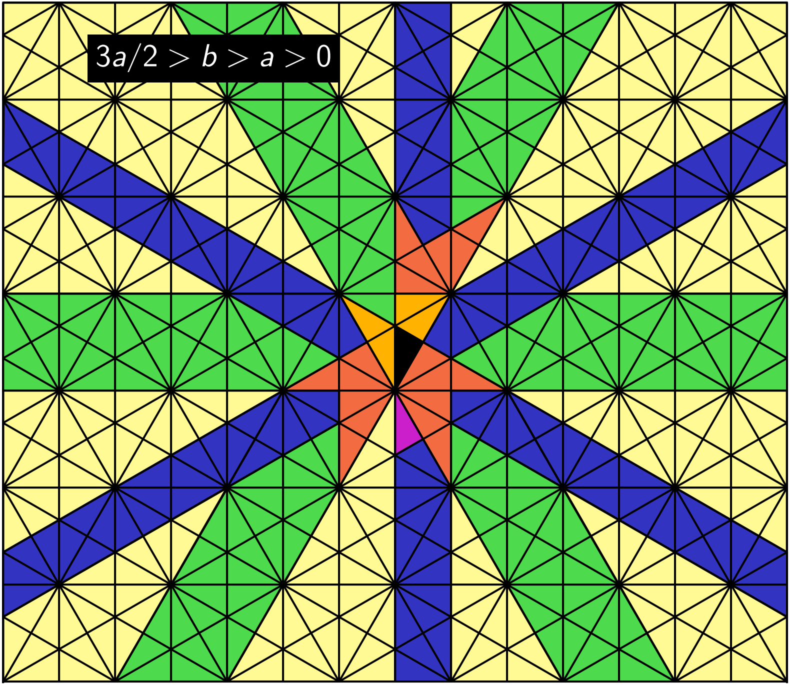
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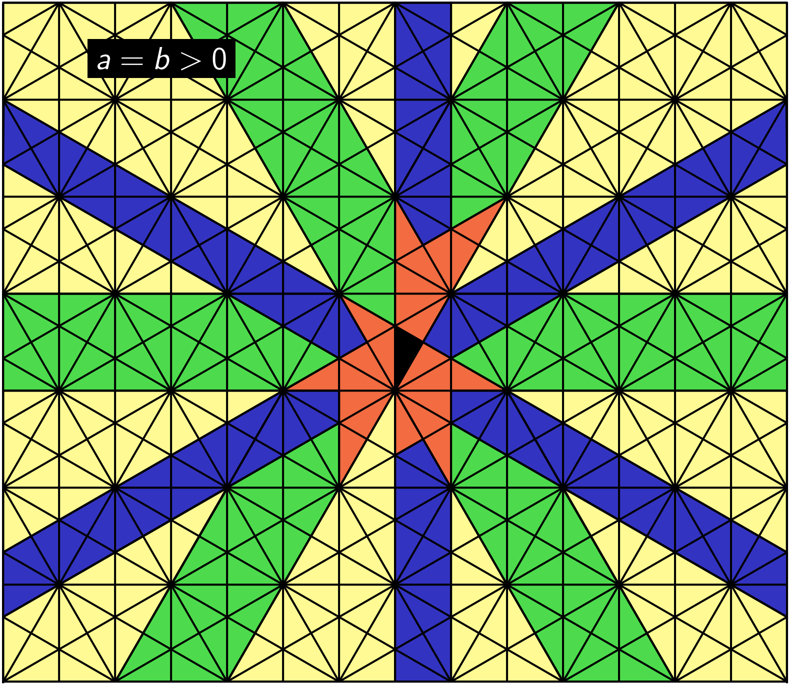
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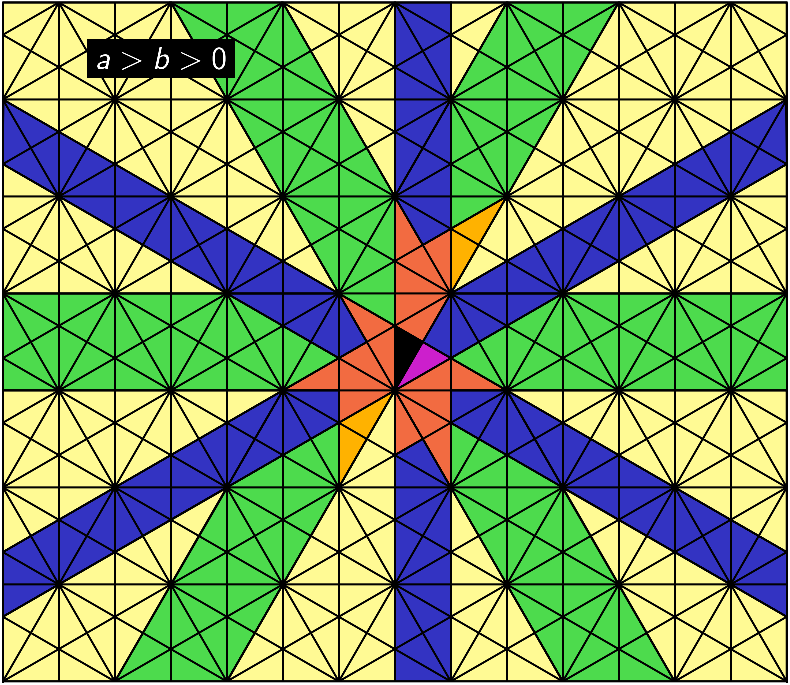
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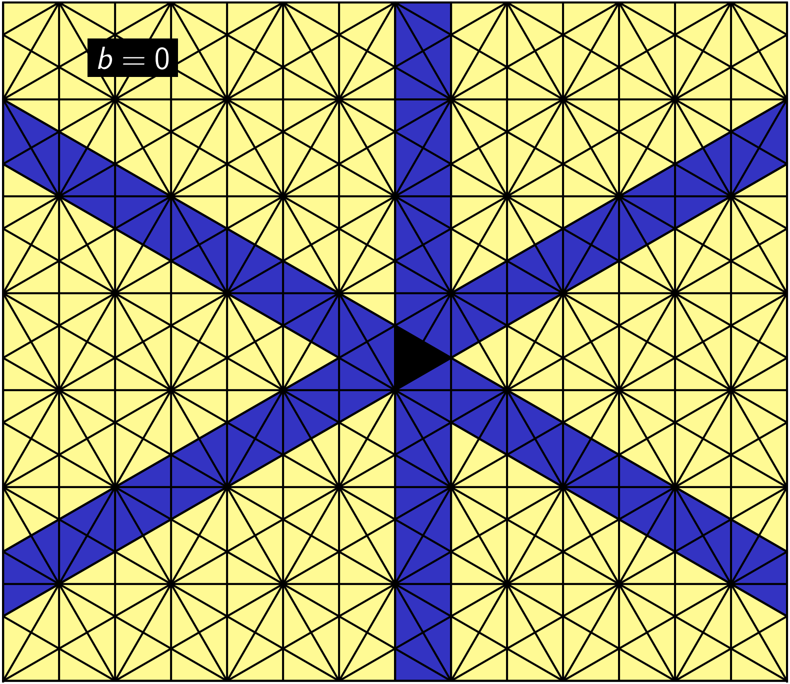
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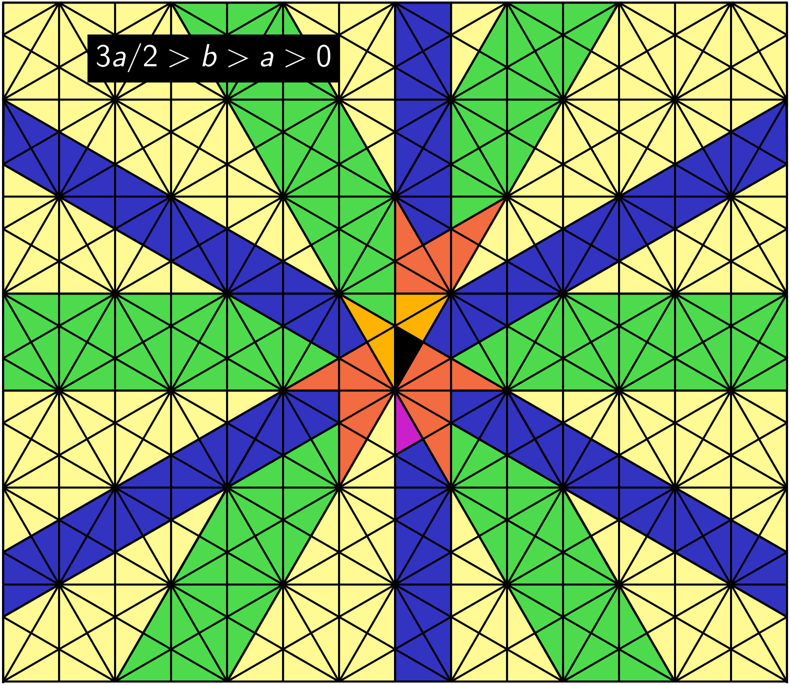
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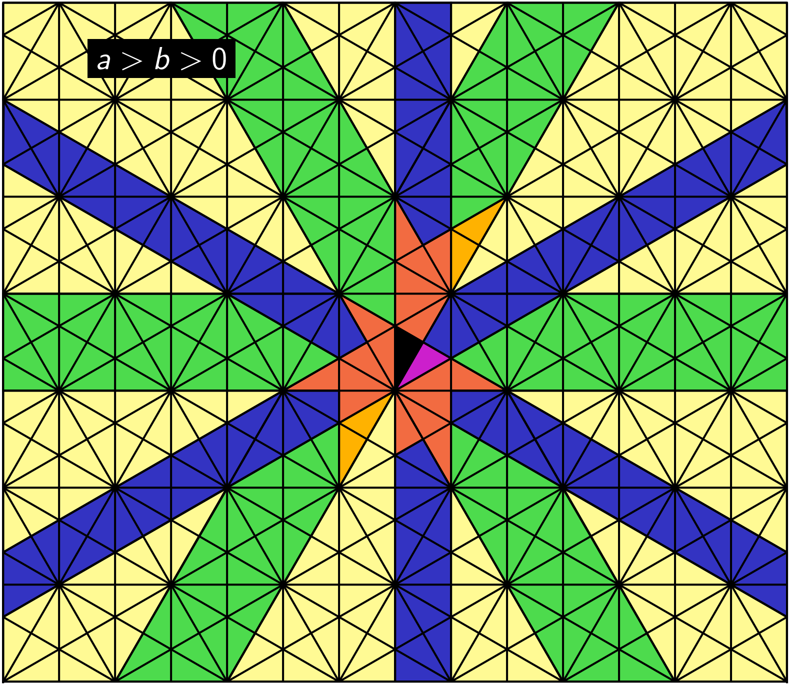
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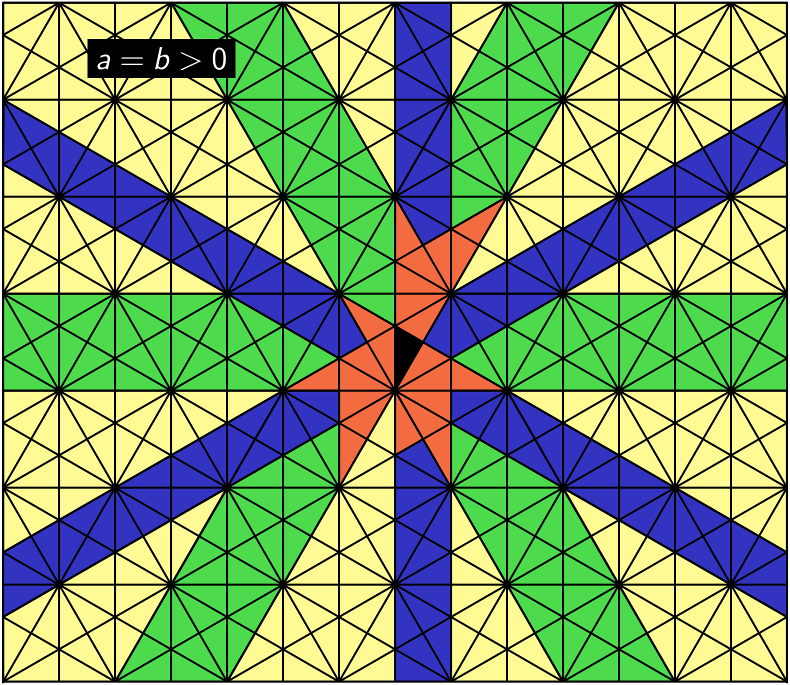
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