Anatoly Vershik - 3/21/08 4pm - "Some problems of Asymptotic Rep^n Theory (ART)"

- introduced by Fred Goodman gives brief bio of the speakers areas & prominent students.

Some Questions:

What happens as rank of rep^n goes to infinity?

Infinite symmetric group? Fourier analysis of this algebra?

Admissible representations?

Consider the infinite symmetric grp $\mathfrak{S}_\infty$.

$\mathfrak{S}_\infty \times \mathfrak{S}_\infty \Big| \supset \text{Diag } \mathfrak{S}_\infty$. Consider $\ell^2(\mathfrak{S}_\infty)$ & the left & right representations ℓ & r. Big group $\mathfrak{S}^\infty$.

How to describe irrep of group $\{(g,h) \in \mathfrak{S}^\infty \times \mathfrak{S}^\infty \mid gh^{-1} \in \mathfrak{S}_\infty\}$?

We know about that, but what about the rep^n's of $\mathfrak{S}_\infty$ alone?

(1964) $\chi_{\alpha,\beta,\gamma}(g) = \prod_{n \geq 2} S_n(\alpha,\beta)^{n(g)}$ $\quad \chi(e) = 1$

where $\alpha = (\alpha_1 \geq \cdots \geq 0)$ $\quad \beta = (\beta_1 \geq \cdots \geq 0)$ & $\sum \alpha_i + \beta_i + \gamma = 1$.

Super newtonian sum $S_n(\alpha,\beta) = \sum \alpha_k^n + (-1)^{n-1} \sum \beta_i$.

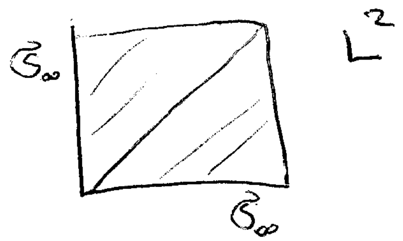
Looking at $\exp \sum_{n \geq 2} \frac{\chi(C_n)}{n} z^n$ generating fcn gives insight into total positivity.

$= e^{\gamma z} \prod_k \frac{1 - \beta_k z}{1 + \alpha_k z}$ where $\chi(0) = 1$.

Okunkov considered limits of operators of type $\frac{1}{n} \sum_{i=1}^{n} (i,k) \cdots O_k$

Consider $\{\varepsilon_i, \delta_i\}_{i=1}^{\infty}$ s.t. $\exists n$ with $\varepsilon_{i+n} = \delta_{i+n}$

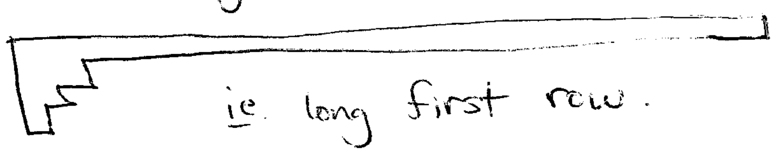
So eventually they coincide. Given $\alpha_1 + \alpha_2 = 1$ & $\alpha_1 \neq \alpha_2$.



What happens if you have $\alpha_1, \ldots$ & $\beta_1, \ldots$?

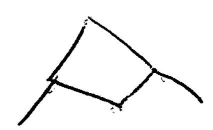
If you have several α's with $\alpha_1 = \cdots = \alpha_n$, Okunkov showed this.

For finitely many α, β where you have multiplicity. Consider the unitary group $U(\alpha, \beta)$. And $\otimes U(\alpha, \beta)$. The rep^n theory of unitary grp is 2 diagrams.

Tensor rep^n looks like diagram  ie. long first row.

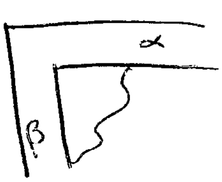
$$\Pi_{\lambda_n} \otimes \Pi_{\mu_n} \longrightarrow \sum$$

Thm The restriction of admissible rep^n to the comp. is the factor-rep's of II_1, II_∞



Invariant or Central measure using Young diag.

Def^n Semifinite measure - the set of the poles is nowhere dense. This measure on Young diagrams - we found



α, β & some diagram where α & β are finite.

Quasiequivalent representations.

What is the quasiequivalence of $\{\vec{\mathfrak{S}}_\infty \times \ell\}$?
$\{\ell \times \vec{\mathfrak{S}}_\infty\}$

$\alpha, \beta \quad \sum \alpha_i + \beta_i = 1$

$\alpha, \beta \quad \Lambda_1^\circ \quad \Lambda_2^\circ \quad \sum \alpha_i + \beta_i = 1$

Consider $\{0, 1, \overset{h_a}{a}, \overset{h_2}{b}, \ldots \overset{h_?}{c}\}$.

<u>Coupling Constant</u> (introduced by Von Neuman)

constant $0 < C < \infty$. $\quad C = \dfrac{tr\{\vec{\mathfrak{S}}_\infty^{\,\prime} f\}^{x}}{tr\{\quad\}^{2}}$

<u>Heisenberg Group</u> $\begin{pmatrix} 1 & * & * \\ 0 & 1 & * \\ 0 & 0 & 1 \end{pmatrix}$ $\mathbb{R}/\mathbb{Z}$.

Consider λ_1 & λ_2 with grp's $\begin{pmatrix} 1 & n\lambda_1 & * \\ & 1 & m\lambda_2^{-1} \\ & & 1 \end{pmatrix}$ & $\begin{pmatrix} 1 & r\lambda_2 & * \\ & 1 & s\lambda_1^{-1} \\ & & 1 \end{pmatrix}$.

These 2 groups compute. Consider the operator $L^2(\mathbb{R})$ & we get factors that are II_1, $\mathfrak{A}_1$ $\mathfrak{A}_2$

Wasserman looked at Kac Moody algebra & when you obtain factors.

If you look at the admissible repⁿ of the infinite Symmetric grp, we know little about the operators on this grp. A result by N. Tsilevich & V. follows:

$$\mathbb{C}[\mathfrak{S}_\infty] \supset GZ$$

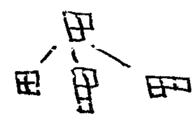
↑ Gelfond Subalgebra

Spectrum of GZ is $T(\mathbb{Y})$ young tableaux.

Two Coincidences:

1) Consider 2 kinds of measures on the set of Young tableaux $T(\mathbb{Y})$.

Markov measure: the probability of next does not depend on previous diagram



2) Induced representation for 2-blocks.

Planchérel measure is dual to Haar measure.

There is plancherel measure on the Young tableaux.

$$\ell^2(\mathfrak{S}_\infty) \simeq L^2(\ ,\ )$$

$Pl\ \ T(\mathbb{Y})$ = plancherel measure on young tableaux

$L^2(T(\mathbb{Y}))$ is an irred rep^n of infinite symmetric grp.

How to describe this rep^n algebraically.

End of Lecture