Supporting the Transition from Arithmetic to Algebra

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Previous work:

- Developing Mathematical Ideas
- Investigations in Number, Data, and Space 2008
- Foundations of Algebra in the Elementary and Middle Grades

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Early algebra

- Generalized arithmetic
  - articulating, representing, and justifying general claims in the context of work on number and operations
- Patterns, functions, and change
  - repeating patterns, number sequences
  - representing and describing contexts of covariation
  - using tables, graphs, symbolic notation

What kinds of arithmetic experiences help preview and build the need for formal algebra?

Aspects of arithmetic experiences that help preview and build the need for formal algebra

- Describing the behavior of the operations
- Generalizing and justifying
- Extending the number system
- Making sense of tables, graphs, and equations
- Understanding notation

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Grade 2 Video Clip
Today’s Number:
25 with Two Addends

In the grade 2 discussion:

- If you change the order, nothing is added or taken away, so the total stays the same (commutativity).
- The teacher and students use cubes to model the action of addition (joining or combining) and to model changing the order of the addends

In the grade 2 discussion:

- The teacher tests the students’ certainty with the generalization by asking them to consider larger, less familiar numbers: “It doesn’t matter. You’re not adding anything or taking anything away.”
- What about subtraction?

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$$\begin{array}{r} 35 \\ -\ \underline{18} \\ 23 \end{array}$$

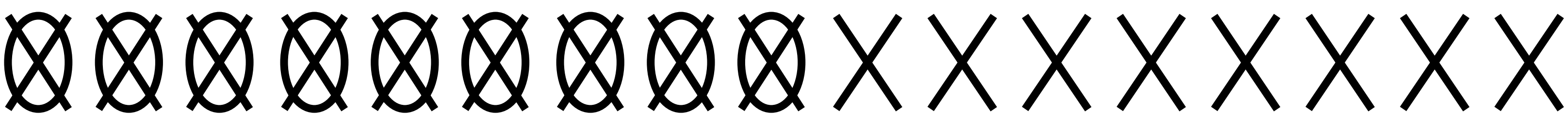
$$17 + 9 = 26$$

$$9 + 17 = 26$$

$$17 - 9 = 8$$

$$9 - 17 = ?$$

9 - 17 = ?



60 - 50 = 10

50 - 60 = invisible 10

$$
\begin{array}{r}
35 \\
-\ \underline{18}
\end{array}
$$

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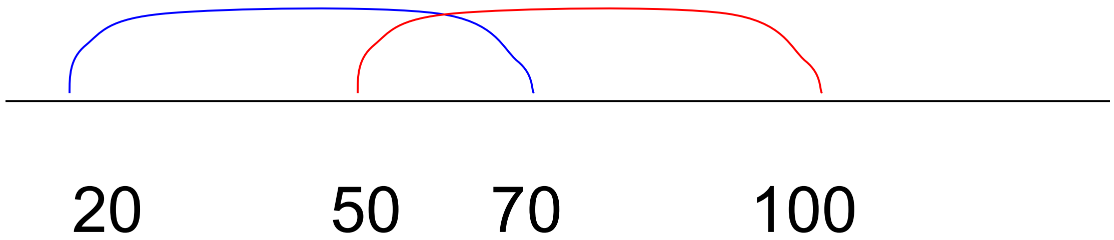
70 - 20 = 100 - 50



You can see that the distance is the same. If you change one number, you change the other the same way. As long as both numbers change the same, you can make lots of new expressions.

100 - 50
90 - 40
80 - 30
70 - 20
60 - 10
50 - 0

Trisha: We could keep adding to our list by changing both the numbers but we are going to get to a point where we won't be able to change the numbers. That will happen when we get to 50.

Nicole: Yes, I agree with Trisha. Because if we look at Alex's number line we are going to get to zero and 50, and the jump will be 50, but then we are done.

Raul: But we could use the other numbers.

Teacher: What other numbers?

Raul: The negative numbers on the other side of zero.

Alex: I have one we can use. Let’s use 40 and negative 10.

Teacher: How do you want me to write that on the chart?

Alex: Put 40, then the subtraction sign and then a negative 10.

$$40 - (-10) = 50$$

Josh: No way, you can’t do that. How can you have a negative 10 and end up with 50?

Alex: It is like adding 10, because if you look on the number line you would have to jump 50 to get from negative 10 to 40. It is the same as we did with 100 and 50 and 70 and 20.

Teacher: So, Alex, how do you know that 40 minus negative 10 will give you 50?

Alex: Because you have to add 50 to negative 10 to get 40.

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Is there a rule for predicting whether the sum is going to be negative or positive when you add a negative and a positive?

SPOKEN: Let N be a negative number and P be a positive number. If N is bigger than P, then N + P equals something negative. If P is bigger than N, then N + P equals something positive.

WRITTEN: Let N be a negative number and P be a positive number. If N > P, then N + P = N; if P > N, then N + P = P.

Is there a rule for predicting whether the sum is going to be negative or positive when you add a negative and a positive?

SPOKEN: The answer will have the same sign as whichever number has the larger absolute value.

WRITTEN: Let N be a negative number and P be a positive number. If $|N| > |P|$, then $N + P < 0$. If $|P| > |N|$, then $N + P > 0$.

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Grade 2 Video Clip: Comparing Contexts

Cube Train

1	2	3	4	5	6	7	8	9	10	11	12

The pattern of YYOOB cubes continues in the same way.

What number is the first blue cube?
What number is the second blue cube?

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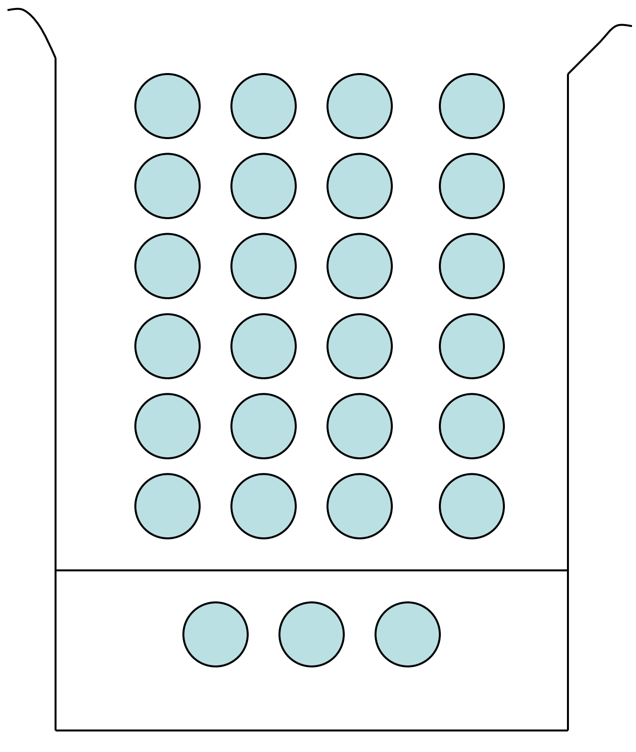
Grade 4 Video Clip:
The Penny Jar



(round #) x 9 + 4 = total number of pennies

Multiply the number of rounds by the number you add each time. Then add the start number.

R x A + S

R x 9 + 4 = number of pennies

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What is needed as a foundation for making the transition to algebra?

- National Math Advisory Panel report:
  - Fluency with whole numbers
    - Understanding of place value
    - Ability to compose and decompose
    - Grasp of the meaning of the operations
    - Use of the commutative, associative, distributive properties
    - Computational facility
    - Knowledge of how to apply operations to problem solving
  - Fluency with fractions

What does it look like when students *don't* have experience with these aspects of arithmetic when they reach algebra?

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$$2\,(a)\,(b) = (2a)\,(2b)$$

$$(a + b)^2 = a^2 + b^2$$

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Trishna, grade 3

Routine: Days in School—finding factors of the number of the day

—you can’t count to any odd number by 2s

—can you count to any odd number by any other even number (4, 6, 8, etc.)?

Teacher: Is it possible that we're going to find an odd number that we could break apart into evens?

Most of the class: NO!

Teacher: No? Never? Even though we haven't tried every odd number in the whole world, have we?

Julia: That would take way too long!

Teacher: So, Julia, I guess that's the point I'm pushing on here right now. How do we know that, even though we haven't tried every single odd number? So, what can we offer as proof of that?

Evan: You haven't built up enough for another two. You only have one more.

Trishna: I'm just saying another reason why I absolutely positively know that if you can't do two's you can't do any even number…It's almost like two IS every single even number, because you can ALWAYS find two in every single even number. So, it's like two is all the even numbers. But, you can't find fours in every single even number.

Teacher: But, if you had fours….If I could break this number today up into four's…

Trishna: You'd also HAVE to have two's. (Emphatically) Two is in every single even number.

Trishna, Grade 3: Finding Factors of the Number of the Day

It gives you a different perspective on the numbers. You start thinking of a number like 88, and then all of these things about a number come to mind. Is it a count of 5? 8?

I have never been in a class where we talked about stuff like that before. It's like taking the number and bringing it alive, making it much bigger. 88 is not just like two 8's next to each other. It's also a count of 11.

Trishna, Grade 3 . . .

We have so many theories, and that's got me thinking. I talk about it at home all the time. This morning, I woke up and told my mom that today was a prime number, and she said, "What?"

When we're all thinking about it together, we have 21 ideas. It's not just me thinking about it for myself.

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