What algebraic understandings do we wish future teachers might gain in college? Beyond what might be essential for success in beginning collegiate mathematics?

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A stimulus for conversation

- An imagined classroom
- With active students
- Asking mathematical questions
- About typical procedures for solving algebra equations
- Not exemplary teacher practice, but representative of what might happen

Blue’s questions from The Great Divide

“What did Red do?”

“Why would you do *that*? I’m not even sure what that means!”

“If we subtracted 5 from both sides, we would subtract 5 from just the 5 and the 65. But here, didn’t Red divide *everything* by 5?”

“I’m convinced Red got the correct solution in *this* case, but does that way always work?”

Questions from The Balancing Act

Purple: So, is 30 the only answer or could there be other ones?"

Orange: "How can you say that it's a scale? When Blue plugged in 5, 10, and 20 it wasn't balanced."

Gray: "I don't get it. How can x be the right number… and at the same time all the numbers that Blue tried?"

Purple: Can we always assume that the equation will have an answer?"

Gray: "So it's okay to treat any equation like it's balanced? Even if it's not?

Questions from the Difference is NoTable

Red: “Well, there aren’t any places where the y-values are the same…I don’t know if I should scroll up or down.”

Yellow: “Would it be okay to solve by Blue’s method whenever I wanted?”

Red: “Can we go over another one using Blue’s method?”

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Introduction:

DC has been running TheMat (Thought experiments in mathematics teaching) project where they have developed some strategies for teachers to talk about "conundrums" of the classroom. The have created animations that can serve as stimuli for conversations. There are some important features of the animation. Firstly, the classroom depicted is an imagined classroom with active students who are asking mathematical questions about typical procedures for solving algebra equations. One needs to note that the practice in the animation is not exemplary teacher practice, but instead is meant to be representative of more typical practice in more typical contexts. The animation is 2 dimensional with a voice-over. It is made for the research project for research purposes. DC notes that one instrumental thing about this approach is that participants can imagine alternative or extended situations to what appears in the animation and they play out what happens in these alternative situations. Since the animation necessarily under-specifies the complexities of teaching, participants are encouraged to reflect on additional relevant information that they would deem necessary in order to respond adequately to the prompts around the animation viewing.

The objective of this parallel session is to generate conversation around the question in the title. The content in the video is very standard math content: solving linear equations.

Short description of what happened in the animation:

The children in the class portrayed in the animation are working on: 20x+5=5x+65. After the students worked quietly, one student came up to the board. The teacher asks a student Blue about what Red did. [Red has first factored out a 5 and is solving the equation 4x+1=x+13]. Blue is confused because he thinks he recalls that they should always add/subtract first. One student asks: "If we're allowed to divide at the end, shouldn't we be able to do it at the beginning?" *At the end of the session, Red demonstrates that his approach was valid by substituting his solution into the reduced equation and verifying.* The teacher asks Blue if Red has convinced him. Blue responds that he's convinced Red got the right answer, but wonders if it will always work. Another student, Green, volunteers to go to the board and he verifies that the solution that Red found works in the original equation also.

Discussion of the

In the previous 3.5 minute clip, there were a series of question. The questions that Blue asked were:

- What did Red do?
- Why? What does it mean?
- We do we subtract 5 from just numbers the numbers, but divide everything by 5?
- Does it always work?

Question to the audience: Are these important questions? Choose a question that is important and think about how you would have liked a teacher to have responded. What knowledge do you think is required to respond to these kind of questions?

Noticings about and discussion of the animation:
We discussed these questions in groups and then reconvened for a discussion of the videos.

The teacher honors the students' questions as being valid. When she asks "But is there anything wrong with Red did?" she gave authority back to red to justify his thinking. Some suggested that would want the class to unpack the two statements solution approaches side by side.

The situation in the clip provided the opportunity to discuss multiple solutions and show how they are equivalent and why/when you might choose a particular approach.

People wanted to know what happened the day before this clip. What are we working from? What do the kids know from their previous experiences in class?

Sybillia's comments. It was hard to pick a single question. Instead she'd like to address key ideas. The distributive property was obviously a key confusion for the student who didn't see how the two solution methods were equivalent. Another issues that you'd want students to understand is what operations you're allowed to perform in order to get equivalent equations. A third big idea raised by the clip is what qualifies as a convincing argument for why it is correct or why a statement is always true (as opposed to giving evidence for the truth of a statement).

A comment on one piece of prior knowledge that we see evidenced in the tape—The students say that they can subtract 5 from both sides. Do they have a model (e.g. scale balance model) in mind for what operations on equations mean? Can that same model be helpful to them in solving the problem?

Someone noted the way the teacher at every opportunity threw the question back to the class [well to Red in particular].

Steffen commented on the importance of helping students to see the difference between "What you can do" versus "what might be advantageous to do in the context of solving a particular problem." He'd want students to be aware of the variety of "legal" operations available to them in transforming the equations. What he called a "do no harm" principle.

Again the issue of the videos being underspecified arose in conversation. Dan reminded everyone that one aspect of the animations that has been useful for figuring out how teachers understand their practice has been that one can project themselves into the situation in the animation. When things are missing or undetermined, one can ask what information one would need in order to make sense of the situation in the animation.

Someone points out that the two students may be asking/answering completely different questions. The second student may be thinking about two numbers multiplied together that have a common factor. It is not clear that Blue is thinking this way.

Mark Saul on how the teacher should respond to the third question that Blue raised. He felt that the response should not just be at the level of the distributive law.

Conversation: What does this clip tell us about the knowledge teachers might need to have available in order to support the development of student understanding as it unfolds in the classroom?

We now discuss "What kind of teacher knowledge needed to respond in this way?" Someone comments that of course mathematically, you can produce an explanation, but mathematically correct explanation is not always the same thing as a pedagogically sound explanation. Some people suggest that you'd want teachers to know that it might be helpful for the students to draw a picture of the situation [e.g. balanced scale model] and use the picture to think about what is happening as one operates on the equations.

Someone suggests that it might be helpful for the teacher to develop the idea of the distributive law more concretely through the use of manipulatives. It might be more meaningful for students to have a concrete manifestation of why the two equations are equivalent.

Someone suggested having students abstracting that the distributive law held from doing several numerical examples.

Mark Saul commented that he would NOT do that. Look at $x+1=6$. Then subtract from each side. Use blue's devil's advocate argument. Then do increasingly diff things. Then thinking of form of algebra. Do at an algebraic level rather than an arithmetic level.

But back to the question of what kind of knowledge do we want teachers to have?

Someone offers that Green might have some of the knowledge that we're thinking the teacher might need. For example, it would be useful if the teacher understood and communicated to students that we're subtracting 5 from BOTH SIDES not just focusing on the individual 5. It seems like Blue is confused exactly at this point: he is not realizing that we really are subtracting 5 from both sides (and not just the "numbers" on each side).

Someone felt that in order to answer the question about what teacher knowledge would be necessary, we'd need to have a better idea of the objective of the lesson. The teacher would have to decide where they wanted to get with students that day and this would guide how they would deal with questions that came up.

Someone reflected that in terms of teacher knowledge, the question is whether there is knowledge of how to solve an equation that a teacher needs to have that is greater than just how to actually go through the steps of solving.

Jo Ann Lobato commented that teachers need to be able to reason quantitatively not just in terms of equation and equivalence preserving operations. For example, they should be able to use a discrete model and be able to describe WHY each operation works in terms of the discrete model. Teachers need to be able to recognize that 1) it's possible to approach this way and 2) it would be beneficial to the class to have an extended discussion around a model. You would want a teacher to be able to think about the thought processes of their students that led to an incorrect response.

Someone pointed out (again in connection to the under-specification of the video) that knowledge of how to appropriately handle the situation is connected to the teachers' knowledge of his or her students. For example, the interaction with Blue could be very atypical and might signal to the teacher that many other students are probably confused about the same things. However, the interaction we saw might be typical of Blue and would signal that a different class-level response would be appropriate.

The session ended with a short recap that the point of the session was to think about what teachers need to know in order to support the development of algebraic thinking in their students. What do they need to know to be able to answer their students' questions and understand things that are confusing to them. How do we have a system where teachers are adequately prepared for this need? What can we do in college to support this? The animations shared in this session in the context of a middle school math content course might provide a useful provocation to start thinking about questions that will arise in the professional lives of teachers.