

Jim Fowler: Lattices with torsion and rational homology manifolds

Question (Wall)  $B\Gamma$  P.D.  $\stackrel{?}{\rightarrow}$   $B\Gamma \cong \text{mfld.}$  "Existence Borel conj."

Conjecture (B.F.M.W)  $B\Gamma$  P.D.  $\rightarrow B\Gamma \cong \mathbb{Z}$  homology mfld.

Q. (Davis)  $B\Gamma$  P.D. over Ring  $\mathbb{R} \stackrel{?}{\rightarrow} B\Gamma \cong \mathbb{R}$ -homology mfld.  
 $\Gamma$  torsion free.

Related Question:  $R = \mathbb{Q}$   $\Gamma$ -torsion

$X$  space,  $\pi_1(X) = \Gamma$   $\tilde{X}$  (univ. cover) —  $\mathbb{Q}$ -acyclic and  $\mathbb{Q}$ -homology mfld.

Examples:  $\Gamma$ -uniform lattice.

$\Gamma$  torsion free  $K\backslash G/\Gamma$   $\mathbb{Z}$ -P.D.

$\Gamma$  not torsion free  $B\Gamma$   $\mathbb{Q}$ -P.D.

<4.

$\pi_1(K\backslash G/\Gamma) \neq \Gamma$   
 $\mathbb{Q}$ -homology mfld, singular

Thm:  $\Gamma$  uniform lattice,  $\Gamma$  with torsion ( $\mathbb{Z}$ -torsion?)

$\rightarrow \exists X$   $\pi_1(X) = \Gamma$ ,  $\tilde{X}$   $\mathbb{Q}$ -acyclic  $\mathbb{Q}$ -homology mfld.

Examples exist for  $\Gamma$  (not lattice) with torsion st.  $\exists X \dots$

Finikens:

Convenience:  $\Gamma_0 \triangleleft \Gamma$  torsion free finite index subgroup with  $\Gamma/\Gamma_0$  cyclic

$$B\Gamma_0 = K\backslash G/\Gamma_0 \supseteq \mathbb{Z}_p$$

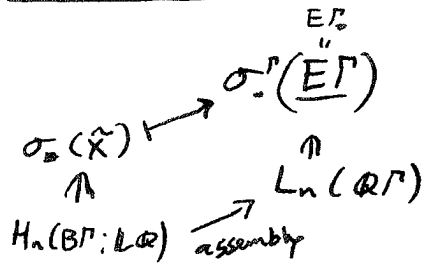
Q:  $\exists X$   $\pi_1(X) = \Gamma$   $\tilde{X}$   $\mathbb{Q}$ -acyclic,  $X$  finite complex

Letzsch:  $\chi\left(\left(\tilde{X}/\Gamma_0\right)^{\mathbb{Z}_p}\right) = 0$

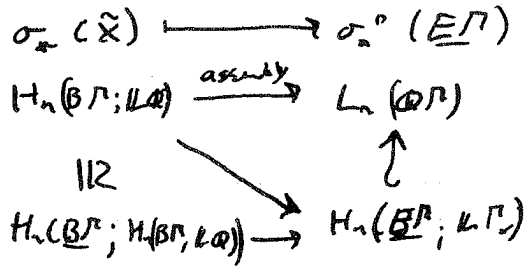
$\chi(B\Gamma_0^{\mathbb{Z}_p}) = 0$  — necessary condition.

Necessary and sufficient:  $X$  (each connected comp. of fixed set)  $= 0 \rightarrow X^{\text{equiv.}} = 0$

This is very doable eg odd dim or cross with  $S^1$ .



if we had such an  $\tilde{X}$ :



So, to prevent existence, make sure  $\sigma_n^*(E\Gamma)$  not hit by diag. arrow.