MSKI 10th Sept. 08 Topology of stratified spaces

Daniel Matei: Cohomology of complements to algebraic plane curves

(Joint with J. Cogolludo)

Problem $V \subset \mathbb{P}^n$ $M = \mathbb{P}^n \setminus V$
alg. hypersurface

Cohomology algebra $H^*(M; \mathbb{Q})$

For class of hypersurfaces: hyperplane arrangements

V = union of hyperplanes $\leftarrow H = \{f = 0\}$

cohomology ring $H^*(M; \mathbb{Z})$ — generated in degree 1 by logarithmic 1-forms — $\frac{df}{f}$

$\updownarrow$

Lattice of intersections of hyperplane arrangement V.

generally: M - 1 formal (formal space)

2 - dim : curves $\mathcal{C} \subset \mathbb{P}^2$, $M = \mathbb{P}^2 \setminus \mathcal{C}$

$H^*(M; \mathbb{C})$ = ? what ~~type~~ type of data does it depend on?
algebra

"Combinatorics" of $\mathcal{C}$: $(T(\mathcal{C}), \mathcal{C})$
tubular nbhd.

Combinatorial type of $\mathcal{C}$
$K(\mathcal{C})$

* - set of irred components $\mathcal{C} = C_0 \cup C_1 \cup C_2 \cup \dots C_r$ and their degrees $d_0, d_1 \dots d_r$

- set of singularities $\mathrm{Sing}(\mathcal{C})$ ~~...~~ and their topological type. $\Sigma = \{L_p\}$ - links of $p \in \mathrm{Sing}(\mathcal{C})$

* - pairwise intersection numbers among branches at p

- with degrees, determines genera of comp. of $\mathcal{C}$

* - incedences ~~of~~ among local branches at $\mathrm{Sing}(\mathcal{C})$ and the global components

Δ_p - set of branches at p

$\phi_p : \Delta_p \to \{0, 1, \dots, r\}$

$W(\mathcal{C})$ - weak combinatorial type *

<u>Theorem:</u> $H^*(M;\mathbb{C})$ cohom. alg. depends only on W.

- Explicit presentation for alg. $H^*(M;\mathbb{C})$ using poincaré residue operators.

Brieskorn Lemma: $H^*(M;\mathbb{C}) \hookrightarrow \mathcal{E}^*(M,\mathbb{C})$

diff. forms.

<u>Cor.</u> M is formal

still to add:
$b_i(M)$ depend only on degrees and on genera

<u>Rem.</u> A. Macinic :

For simplicity $b = C_0 \cup C_1, \ldots, \cup C_r$, $C_0 = \{z=0\}$ and

C_0 transverse to $b \setminus C_0$

<u>deg 1</u> $H^1(M,\mathbb{C}) =: E^1$

E^1 generated by logarithmic 1-forms

$$\sigma_i = d\left(\log \frac{c_i}{c_0^{d_i}}\right) \qquad 1 \le i \le r$$

<u>deg 2</u>

$$\underset{\text{log part}}{E^2} \oplus \underset{\substack{\text{hdom.}\\ \dim g}}{K^2} \oplus \underset{\dim g}{(K^2)^\perp} = H^2(M,\mathbb{C}) \qquad g = \sum_{c_i} \text{genera}$$

$E^2 =$ 2 forms

$$\left\{ \psi_p^{\,\delta_1,\delta_2} \right\}, \left\{ \psi_\infty^{\,i,K_i} \right\}$$

$$\psi_p^{\delta_1\delta_2} + \psi_p^{\delta_2\delta_1} = 0 \qquad \psi_p^{\delta_1\delta_2} + \psi_p^{\delta_2\delta_3} + \psi_p^{\delta_3\delta_1} = 0$$