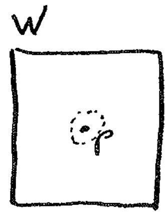
MSRI 10th Sept. '08 Topology of stratified spaces

Mikhail Mazin: Leray-type Operators for Stratified Spaces and the Parshin's Reciprocity Law for Residues



$\mathrm{res}_p W$

$$X = X_n \supset X_{n-1} \supset \dots \supset X_0 \qquad \dim X_k = k$$

(1) assume, all X_i's normal.

$X_k \supset X_{k-1}$

$\exists f_k$ meromorphic function on X_k, s.t. f_k has order 1 at a gen. point of X_{k-1}

$\rightsquigarrow (f_n, \dots, f_1)$

Let ω be a meromorphic n-form on X

$df_n, df_{n-1}, \dots, df_1$ are lin. indep at gen. point of X_{n-1}

$\rightsquigarrow \quad \omega = g\, df_1 \wedge \dots \wedge df_n$

$g = \sum_{k > N_n} g_k f_n^k \qquad \omega_{n-1} = g_{-1} df_1 \wedge \dots \wedge df_{n-1}$ is an $n-1$ meromorphic form on X_{n-1}

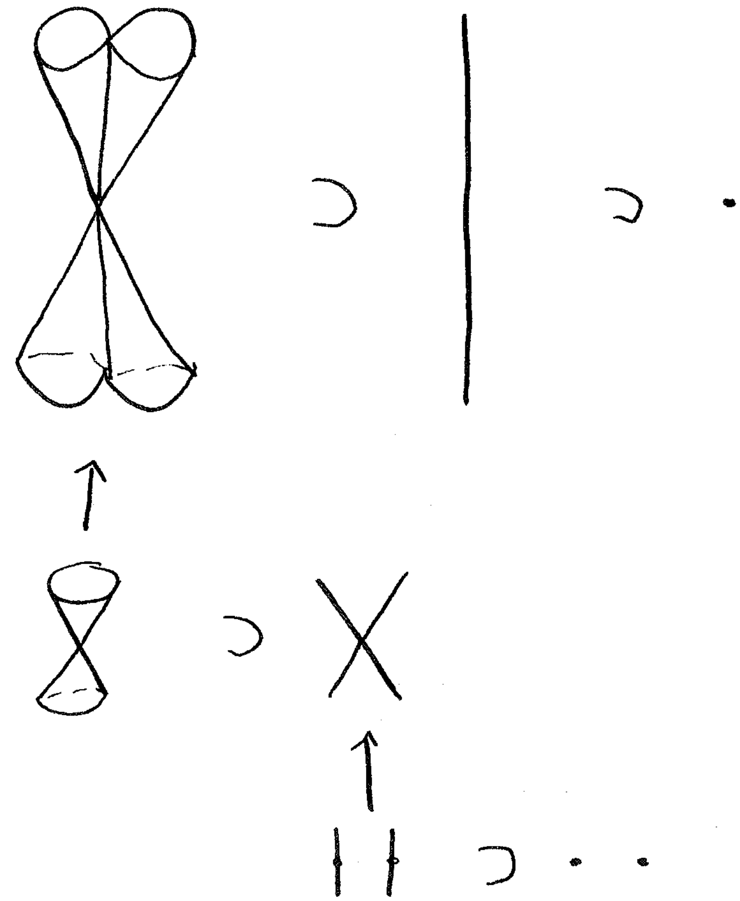
$\dots \quad \omega_0 = \mathrm{res}_F(\omega) \quad F = \{X_n \supset \dots \supset X_0\}$

(2) X_k not normal

$X_n \supset X_{n-1} \supset \dots \supset X_0$ (normalise)

$$\begin{array}{l} X_n \\ \uparrow \pi_n \\ \tilde{X}_n \supset W_{n-1} \\ \qquad\quad \uparrow \\ \qquad\quad \tilde{W}_{n-1} \supset \dots \quad W_1 \\ \qquad\qquad\qquad\qquad\quad \uparrow \\ \qquad\qquad\qquad\qquad\quad \tilde{W}_1 \supset W_0 = \{\text{set of points}\} \end{array}$$

example:



(at each point we have residue)

$\mathrm{res}_{F_{1\alpha}}(\omega)$ defined.

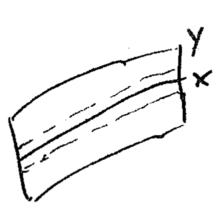
$$\mathrm{res}_F\, \omega = \sum_{\alpha \in W_0} \mathrm{res}_{F_{1\alpha}}\, \omega$$

Thm: (Parshin) $X_n \supset X_{n-1} \dots \supset X_k \supset X_{k-1} \supset \dots \supset X_0$

consider all possible X_k s.t. $X_{k-1} \subset X_k \subset X_{k+1}$

then only finitely many flags will give non trivial residues (for fixed ω) and the sum is zero.

X^n, Y^k two oriented strata of stratified space s.t. $X \subset Y$ and $\nexists Z$ $X \subset Z \subset Y$



$$\varphi_{X,Y} : H_*(X) \longrightarrow H_*(Y)[k-n+1]$$

Thm X, Y strata, $Z_1, \dots, Z_m$ all strata s.t. $X \subset Z_i \subset Y$, Z_i, Z_j not comparable

$$\varphi_{Z_1,Y} \circ \varphi_{X,Z_1} + \dots + \varphi_{Z_m,Y} \circ \varphi_{X,Z_m} = 0$$