Robert Waelder – Singular Elliptic Genus of Normal Surfaces

Mirror pair $(M, \hat{M})$ complex mflds of compl. dim. n
Top. Mirror Symmetry test
$$h^{p,q}(M) = h^{n-p,q}(\hat{M})$$

$$X \xrightarrow{f} \hat{M} \qquad \text{(crepant) resolution}$$
$$f^* K_{\hat{M}} = K_X$$

in case a crep. resol. exists, take X instead of $\hat{M}$

$$H^i_c(\hat{M}, \mathbb{C}) \rightsquigarrow \text{Deligne mixed Hodge structure}$$
in general $h^{p,q}(X) \neq h^{p,q}(\hat{M})$

Batyrev

$$E(X; u, v) = \sum h^{p,q}(X)\, u^p v^q$$

expand the working category $(X, D) \xrightarrow{f} (Y, D')$
$$f^*(K_Y + D') = K_X + D$$

$D = \sum_I a_i D_i \subset X$ (assume X is smooth)

$$\mathring{D}_J := \bigcap_J D_j \setminus \bigcup_{J^c} D_j \rightsquigarrow E(X, D) \hat{=} \sum_{J \subset I} E(\mathring{D}_J; u, v) =$$

$$= \prod_J \frac{uv - 1}{(uv)^{a_j+1} - 1}$$

(motivated by motivic integration theory

Assume above : $a_i > -1$ $(a_i \neq -1)$ (log terminal singularities)

$$X, D \xrightarrow{\text{log res.}} Z$$

Assume Z $\mathbb{Q}$-Gorenstein log term sing.

$\rightsquigarrow E_{st}(Z) = E(X, D)$ will be independent of small resol. blowups

Similar question

Chern numbers ? Elliptic genus

$$Ell(X; z, \tau) = \int_X \prod_{TX} \frac{\theta(x_i, -z, \tau)}{\theta(x_i, \tau)}$$

$$\theta(t, \tau) \sim \sin\frac{t}{2} \prod_n (1 - tq^n)(1 + t^{-1}q^n)$$

Borisov, Libgober, Chin-Lung Wang , $\sum_I a_i D_i$, $a_i > -1$

$$\int \frac{\prod x_i \theta(x_i - z)}{\theta(x_i)} \prod_I \frac{\theta(D_i - (a_i + 1)z)}{\theta(D_i - z)} \frac{\theta(z)}{\theta((a_i + 1)z)}$$

this relative version is independent of blowups

McKay Correspondence

$G \circlearrowright V$ group

$$\begin{array}{ccc} & & \widetilde{V/G} \\ \varphi^* K_{V/G} = K_V & & \downarrow \text{crepant} \\ V & \longrightarrow & V/G \end{array}$$

data of (V, G) as orbifold should correspond to data of $\widetilde{V/G}$

e.g. $e_{orb}(V,G) = \frac{1}{|G|}\sum_{gh=hg} e(V^{g,h})$

$$\| \\ e(\widetilde{V/G})$$

in case no crep. res. exists

$$\begin{tikzcd} \hat{Z} \arrow[r] \arrow[d] & Z \arrow[d] \\ V \arrow[r, "\psi"] & V/G \end{tikzcd}$$

How to get rid of the log term assumption

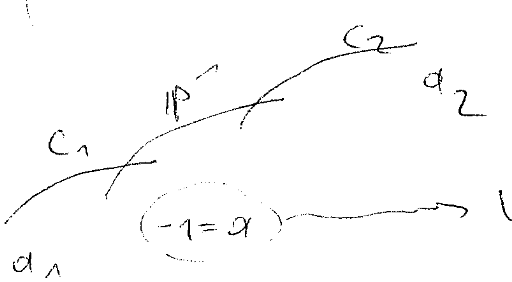
$$\begin{tikzcd} X \arrow[d, "f"'] \\ Z \end{tikzcd} \qquad \sum a_i D_i$$

take an ample div. crossing the circle

$f^*(K_Z - \varepsilon H)$ ← take the limit (doesn't always exist)

William Veys   normal surface singul. not purely log canonical

$(\Leftrightarrow a_i \geq -1) \rightsquigarrow$ E-fct



→ What contribution would it give to Batyrev's stringy E-fct?

$$\frac{(uv-1)^2\left((uv)^{m(a+1)}-1\right)}{\left((uv)^{a+1}-1\right)\left((uv)^{a_1+1}-1\right)\left((uv)^{a_2+1}-1\right)}$$

W. generalize elliptic gen. for normal (sing.) surfaces
→ generalizes Borisov-Libgober and
William Veys

$$\sum_{a_i \neq -1} a_i D_i + (-1) D_0$$
$$\overset{\shortparallel}{(1)(-D_0)} \quad \rightsquigarrow \text{„change roles"}$$

Θ

$$\int \prod \frac{x_i \Theta(x_i - z)}{\Theta(x_i)} \cdot \frac{\Theta(E+2z)}{\Theta(E+z)} \frac{\Theta(z)}{\Theta(2z)}$$

↓

Can show: Borisov-Libgober
ample div. approach
converges to the above
holom. in z

$$c\varphi_{0,1} = E_1 - E_2$$

↑
perturbation term

Show $c=0$