Efficient Data Assimilation for Spatially Extended Systems

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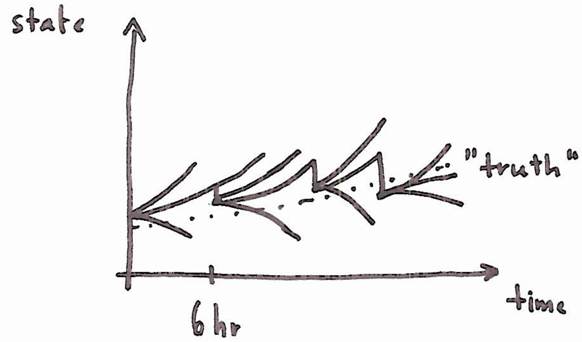

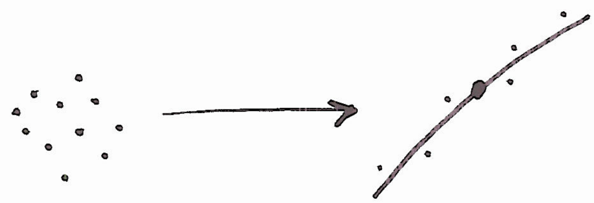

$$x_{n+1} = f(x_n)$$

$$y_n = h(x_n) + \varepsilon_n \quad \text{w/ } \varepsilon_n \sim N(0,R)$$

pdf of y_n given x_n $\sim e^{-\frac{1}{2}(y_n - h(x_n))^T R^{-1} (y_0 - h(x_n))}$

likelihood of x_n given $y_1, \dots, y_n \sim e^{-\frac{1}{2} J_n(x_n)}$

$$J_n(x_n) = \sum_{k=1}^{n} (y_k - h(x_k))^T R^{-1} (y_k - h(x_k))$$
$$\text{w/ } x_k = f^{k-n}(x_n)$$

Problem: Minimize J_n to find the "most likely" value of X_n

Kalman Filter

Assume we've minimized J_{n-1} by "completing the square"

$$J_{n-1}(x_{n-1}) = (X_{n-1} - \bar{X}_{n-1})^T A_{n-1}^{-1} (X_{n-1} - \bar{X}_{n-1}) + C_{n-1}$$

Then do same w/ $J_n(x_n) = J_{n-1}(F^{-1} x_n) + (y_n - Hx_n)^T R^{-1} (y_n - Hx_n)$

$[f(x) = Fx, \; h(x) = Hx]$

$$\boxed{\begin{aligned} \bar{X}_n &= F\bar{X}_{n-1} + A_n H^T R^{-1} (y_n - HF\bar{X}_{n-1}) \\ A_n^{-1} &= (FA_{n-1}F^T)^{-1} + H^T R^{-1} H \end{aligned}}$$

Kalman Filter