

IRREDUCIBLE IMPRECISION IN  
ATMOSPHERIC AND OCEANIC SIMULATIONS:  
IMPLICATIONS FOR CLIMATE SCIENCE  
AND EARTH MANAGEMENT

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## Propositions

1. "All chaotic dynamical systems are structurally unstable."

(Leonard Smith, in conversation)

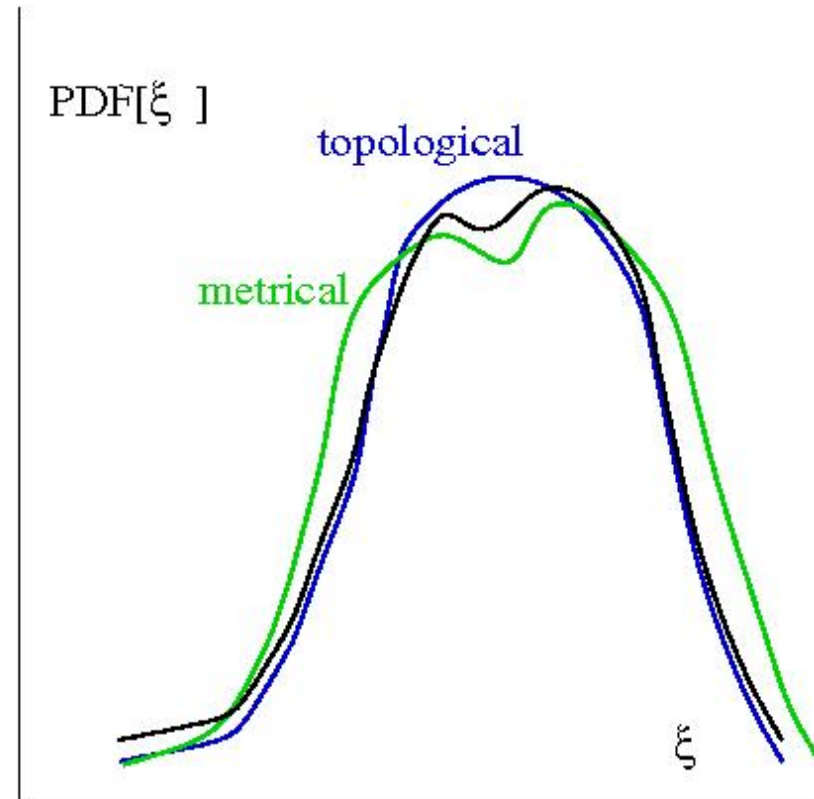
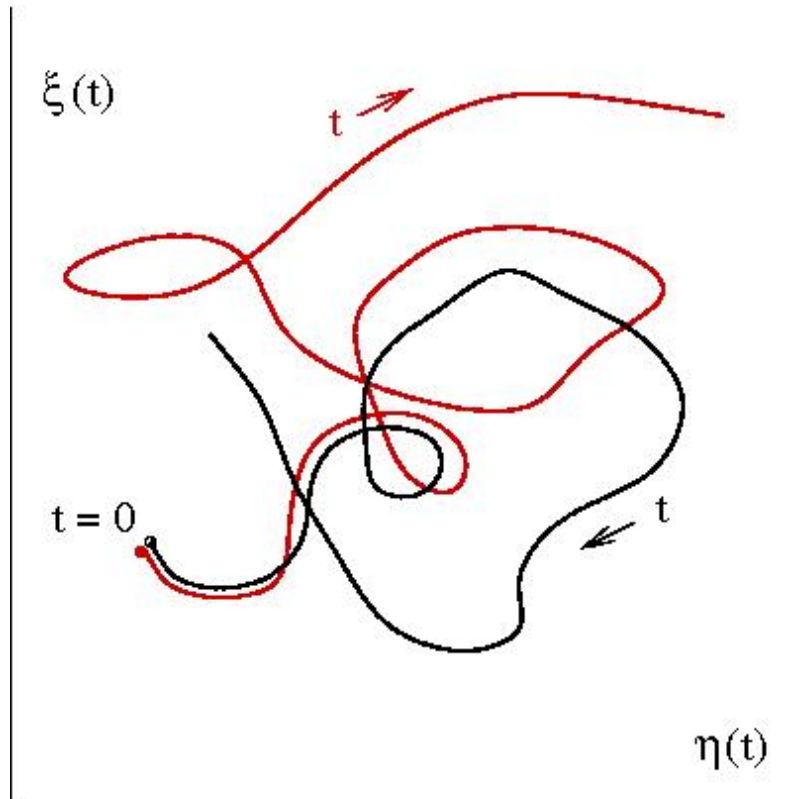
"Structurally stable systems are not dense"

(Stephen Smale, *Amer. J. Math.*, 1966)

DEFINITION: A small change in a model, either for a parameter value or a functional component, leads to appreciable change in the long-time solution behavior (*i.e.*, the attractor), either topological or metrical.

2. Structural instability is untestable in general because there is no meaningful limit for the types of model changes that could be made, only for particular specified types.

Generic behaviors for chaotic dynamical systems with dependent variables  $\xi(t)$  and  $\eta(t)$ : (Left) *sensitive dependence*, small changes in initial or boundary conditions imply limited predictability with (Lyapunov) exponential growth in phase differences, and (Right) *structural instability*, small changes in model formulation alter the long-time probability distribution function, PDF (*i.e.*, the attractor).

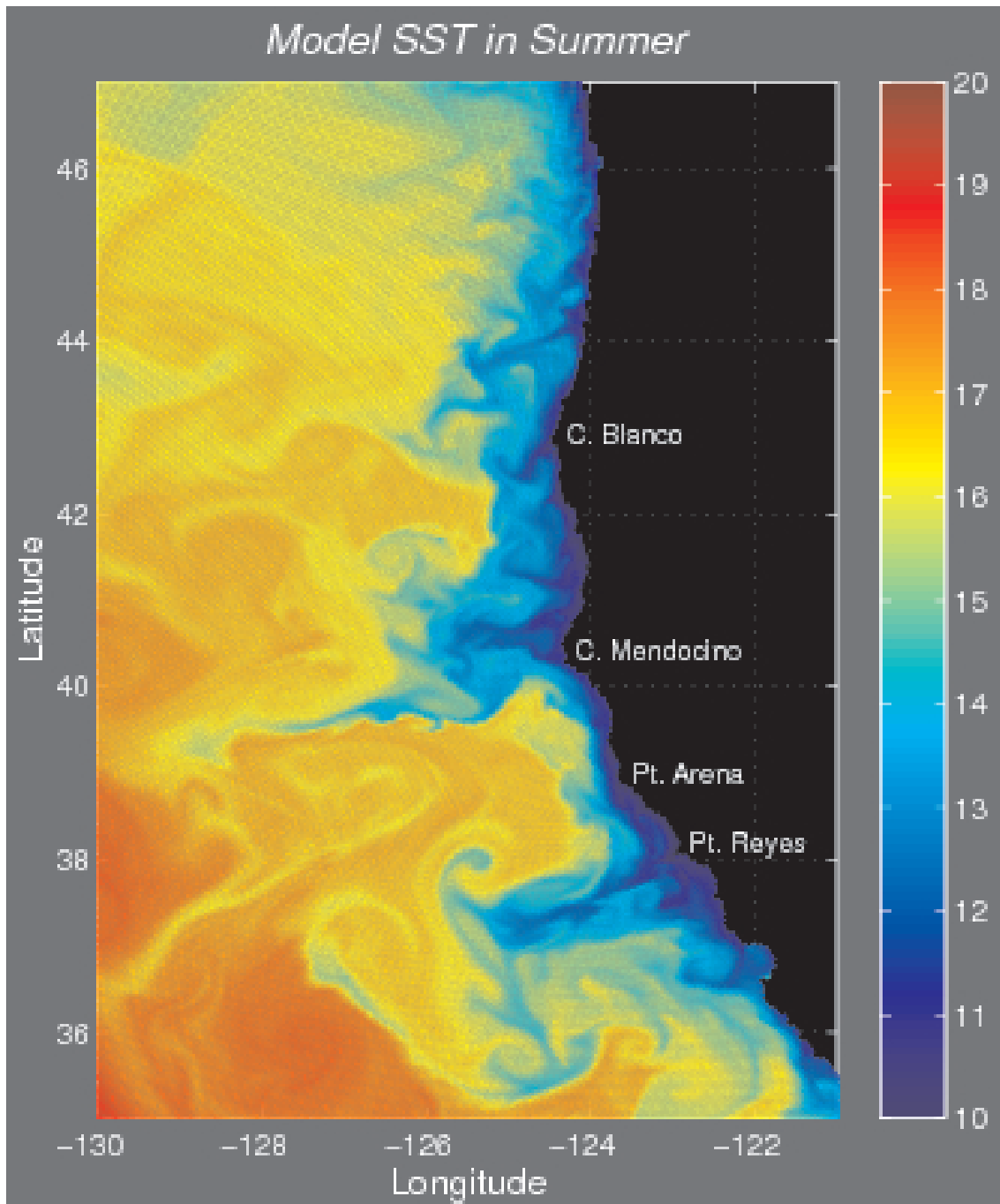


## Propositions (cont)

**3.** Atmospheric and oceanic simulation (AOS) models — turbulent fluid dynamics plus ... — are our most potent tool for scientific discovery and prediction these days (more so than measurements or fundamental theory), and their interesting solutions are chaotic, hence almost certainly structurally unstable.

**4.** AOS solutions are remarkably like nature in many ways, both qualitatively and semi-quantitatively.

⇒ AOS models can teach us about nature and make predictions that can be partially right but will remain partially uncertain.



Instantaneous field of sea surface temperature off the coast of California. Notice the cold boundary upwelling, mesoscale eddies, fronts, and filaments. (Marchesiello *et al.*, 2003)

## Propositions (cont)

5. The climate dynamical system has an essentially limitless number of important contributing processes, hence an enormous variety of plausible model forms and opportunities for structural instability.
  
6. Climate model formulation at a macroscopic system level forces scientists to invent many of the equations with only broad empirical and theoretical constraints — certainly non-uniquely.

Approximate macro-physical dynamics determined by art keyed to experiment, rather than fundamentals and first principles.

“The theory of everything” (Robert Laughlin & David Pines, *PNAS*, 2000)

⇒ We should expect a degree of irreproducibility among AOS model results and imprecision in their simulations of nature [*cf.*, the non-shrinking model spread among global warming forecasts for  $\sim 25$  years].

## Sources of Irreducible Imprecision

- AOS solution fields are non-smooth near the space-time discretization scales (*i.e.*, the “resolution” of the model) imposed on the known governing principles expressed mostly as partial differential equations.
- AOS models contain essential parameterizations for unresolved or highly simplified processes whose specifications are not at a fundamental level of known governing principles.
- AOS models are open-ended in their scope for including and dynamically coupling different physical, chemical, biological, and even societal processes.

## Primary Questions

How can we test the hypothesis of structural instability in IPCC-class and other AOS models?

How can we estimate the level of irreducible imprecision in AOS forecasts and distinguish it from improvable model deficiencies?

How can we communicate the combination of fidelity and imprecision in AOS modeling?



## Coping Strategies

Use of models to study processes and phenomena apart from precise comparisons with nature.

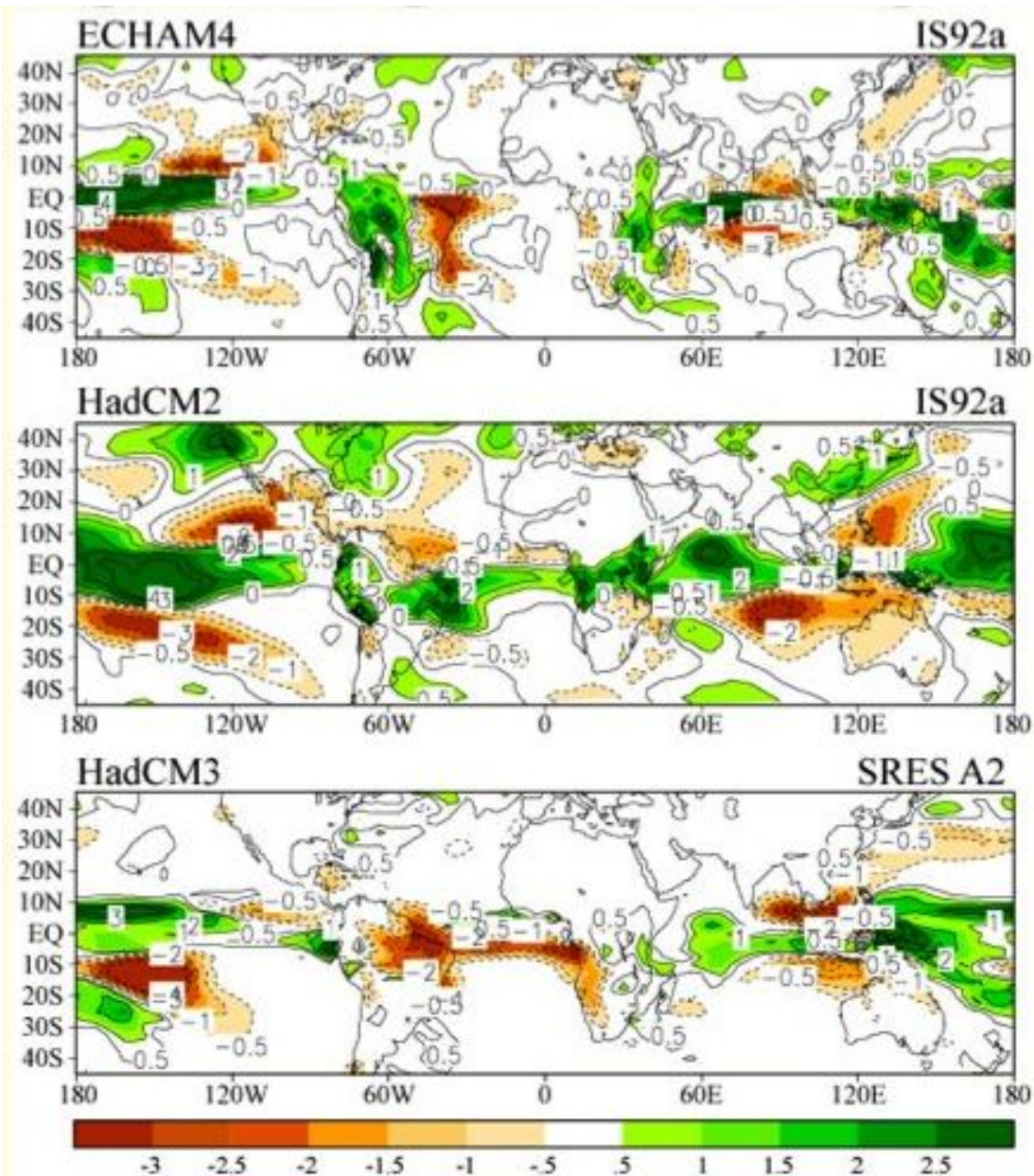
Search for more robust discretization and parameterizations (*e.g.*, differentiable parameterizations?, stochastic PDEs?), as part of the continuing model improvement activities.

Deliberate design of model ensembles (not merely inadvertent and opportunistic, as in IPCC, AMIP, CMIP, *etc.*); *e.g.*, , document “model tuning” to available observational constraints.

Reframe comparisons with nature and climate forecasts in terms of model-ensemble distributions.

(McWilliams, *PNAS*, 2007)

# Precipitation Change Under Global Warming



Predicted average DJF precipitation change [mm day<sup>-1</sup>] between 1961-1990 and 2070-2090 from different climate models with similar mid-range emission scenarios (from Neelin *et al.*, 2003).

Note agreement in the broadest aspects but substantial disagreement in specific patterns and magnitudes.

## Lorenz's Model

Lorenz (1963) is a 3<sup>rd</sup>-order ODE system derived by Galerkin truncation of mid-latitude atmospheric dynamical equations.

As a function of its forcing amplitude  $F$ , it shows successive bifurcations from steady to transient and chaotic states.

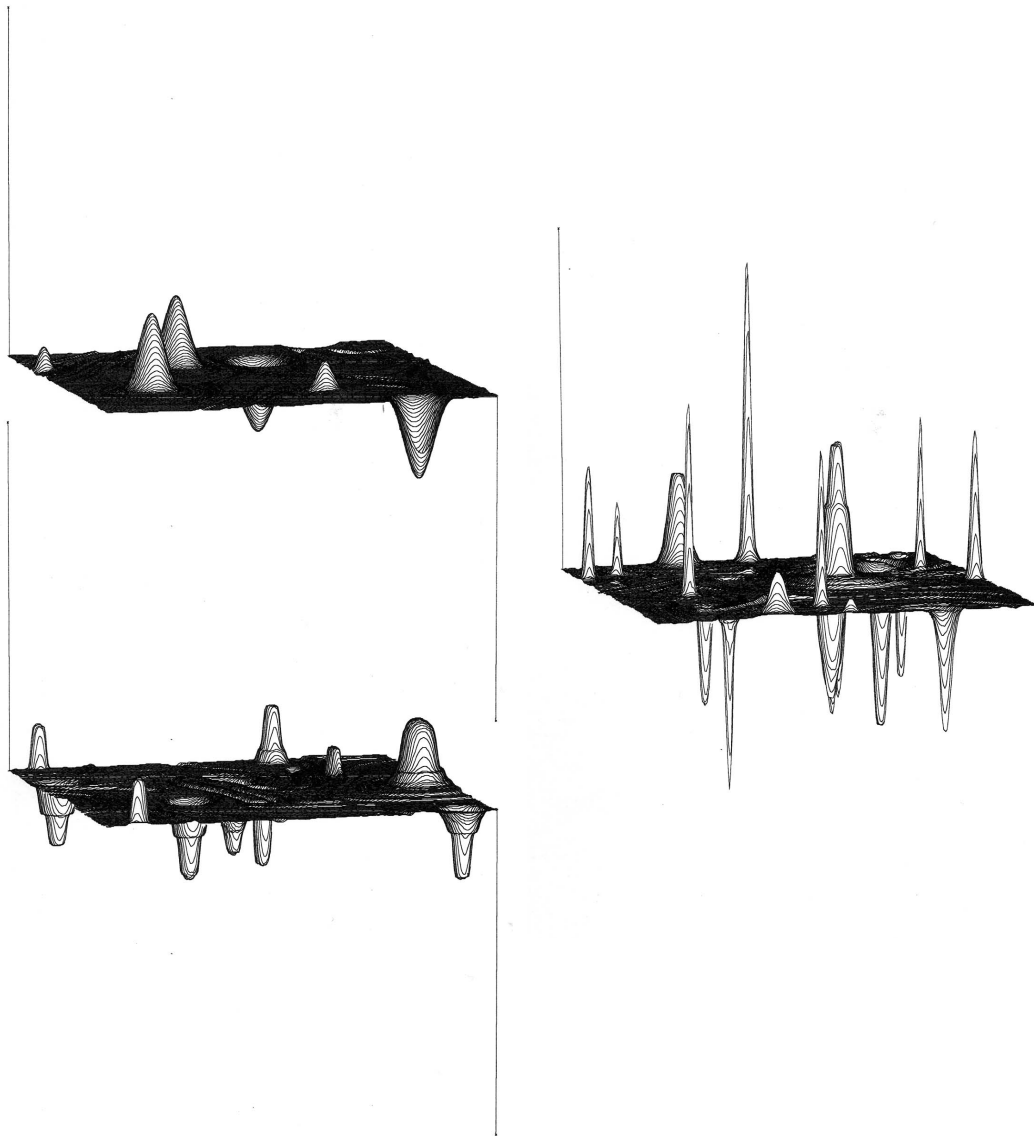
For a certain range in  $F$ , it is famous as a prototype for sensitive dependence and limited predictability.

It also is structurally unstable for a range in  $F$ , since very small  $\delta F$  causes shifts between chaotic strange attractors and periodic limit cycles that are densely intermixed.

This type of structural instability is parametric and topological.

## Fluid Dynamics at High Reynolds Number, $Re$

With the known Navier-Stokes equation and a well-resolved dissipation range  
 $\Rightarrow$  sensitive dependence but structural STABILITY with respect to  $Re$ .



But with alternative choices for "monotone" advection schemes that preserve shape and effect minimal dissipation at a given grid resolution  
 $\Rightarrow$  sensitive dependence & structural INSTABILITY.

Examples of late-time vorticity fields in 2D turbulence using utopia and two varieties of flux-corrected transport (Shchepetkin & McWilliams, 1998).

Numerical algorithms, subgrid-scale parameterizations, and opting for "exciting" (non-smooth) solutions all can induce structural instability of an equation-set and metrical type.