No.
Date.

X : $\mathbb{Q}$-Goren normal variety $/\mathbb{C}$
$Y = \sum t_i Y_i$ ($t_i \geq 0$, $Y_i \subsetneq X$)
$D \subset X$ normal Cartier divisor s.t. $D \not\subset \operatorname{Supp} Y_i$ ($\forall i$)
$\Rightarrow$ $\mathcal{J}(D, Y|_D) = \operatorname{Adj}_D(X, Y)\, \mathcal{O}_D$

Generalize this to higher codim case.

X : smooth variety $/\mathbb{C}$
$Y = \sum t_i Y_i$
$Z \subset X$ closed subvariety of codim k s.t. $Z \not\subset \operatorname{Supp} Y_i$ $\forall i$
$f : \widetilde{X} \to X$ log resol of $(X, Y+Z)$
$E_1, \cdots, E_m$: ex. div on $\widetilde{X}$ dominating a comp of Z
Assume $\sum E_i$ is smooth.
$\operatorname{Adj}_Z(X, Y) := f_* \mathcal{O}_{\widetilde{X}}(\ulcorner K_{\widetilde{X}/X} - f^{-1}(Y) - k \cdot f^{-1}(Z) + \sum E_i \urcorner) \subset \mathcal{O}_X$

<u>Note</u>
$\operatorname{Adj}_Z(X, Y)$ is indep. of the choice of f.

<u>Thm</u>
Assume Z is Goren normal.
$\mathcal{O}_Z \supset J$: l.c.i defect ideal of Z
i.e. $J = \sum_{\substack{Z \subset W : \text{gen'ly l.c.i} \\ \cap \\ X}} \mathcal{H}om_{\mathcal{O}_W}(\mathcal{O}_Z, \mathcal{O}_W)$

Then $\supset$ easier
$\mathcal{J}(Z, Y|_Z + V(J)) = \operatorname{Adj}_Z(X, Y)\, \mathcal{O}_Z$
$\subset$ $\Leftarrow$ We prove this by char. p method.

<u>Example</u>
$0 \in Z = \frac{1}{3}(1,1,1) \hookrightarrow \mathbb{A}^{10} =: X$
Then $J = \mathfrak{m}_{0,Z}^5$ (Kawakita)
$\begin{cases} \operatorname{Adj}_Z(X, 0) = \mathfrak{m}_{0,X}^5 \\ \mathcal{J}(Z, \mathfrak{m}^5) = \mathfrak{m}_{0,Z}^5 \end{cases}$

$(R, \mathfrak{m})$: normal local ring of char. $p > 0$, $\dim R = d$

$X = \operatorname{Spec} R$, $Y = \sum t_i Y_i$

$F^e : \mathcal{O}_X \to F^e_* \mathcal{O}_X$

$\rightsquigarrow F^e : H^d_{\mathfrak{m}}(\mathcal{O}_X(K_X)) \to H^d_{\mathfrak{m}}(\mathcal{O}_X(p^e K_X))$

(1) $N \subset H^d_{\mathfrak{m}}(\mathcal{O}_X(K_X))$ $\quad (q = p^e)$

$\xi \in N \Leftrightarrow \exists c \neq 0 \in \mathcal{O}_X$ s.t. $c \prod_i I_{Y_i}^{t(q-1)} F^e(\xi) = 0$ for ${}^\forall e \gg 0$.

$\tau(X, Y) := \operatorname{Ann}_{\mathcal{O}_X} N$

(2) $Z \subset X$ reduced of pure codim k

$N_Z \subset H^d_{\mathfrak{m}}(\mathcal{O}_X(K_X))$

$\xi \in N_Z \Leftrightarrow \exists c \in \mathcal{O}_X \setminus I_Z$ s.t. $c \prod_i I_{Y_i}^{t(q-1)} I_Z^{k(q-1)} F^e(\xi) = 0$ for ${}^\forall e \gg 0$.

$\tau_Z(X, Y) := \operatorname{Ann}_{\mathcal{O}_X} N_Z$

Thm (Hara - Yoshida, T-) Supp

$(R, \mathfrak{m})$: normal $\mathbb{Q}$-Gorenstein local ring of ess. of f.t $/\mathbb{C}$

$X = \operatorname{Spec} R$, $Y = \sum t_i Y_i$, $Z \subset X$ closed subvar. s.t $Z \not\subset Y_i$

(1) $(\tilde{X}, \tilde{Y})$: reduction to char. $p \gg 0$ of (X, Y)

$\widetilde{\mathcal{J}(X, Y)} = \tau(\tilde{X}, \tilde{Y})$

(2) Assume X is smooth.

$(\tilde{X}, \tilde{Y}, \tilde{Z})$: red to char. $p \gg 0$ of (X, Y, Z)

$\widetilde{\operatorname{Adj}_Z(X, Y)} \supset \tau_Z(\tilde{X}, \tilde{Y})$

$= \leftarrow$ conj.

prop

$(R, \mathfrak{m})$: regular local ring of char. $p > 0$

$X = \operatorname{Spec} R$, $Y = \sum t_i Y_i$, ~~$Z \subset X$~~

$Z \subset X$ Gor reduced of pure codim k s.t $Z \not\subset Y_i$

$W \subset X$ pure codim k scheme of s.t $Z \sim W$ (geom). CI-linked

i.e. (Z and W have no common comp
($Z \cup W$ is C.I

$\Rightarrow \tau(Z, W|_Z + Y|_Z)$

$\subset \tau_Z(X, Y) \mathcal{O}_Z$

This implies $\mathcal{J}(Z, Y|_Z + V(J)) \subset \mathcal{I}/_{Z}\, \mathrm{Adj}_Z(X, Y)\mathcal{O}_Z$

$$\begin{tikzcd}
0 \arrow[r] & \mathcal{O}_X \arrow[r] \arrow[d] & \cdots \arrow[r] & \mathcal{O}_X \arrow[r] & \mathcal{O}_X/I_{Z\cup W} \arrow[r, phantom, "="] \arrow[d] & \mathcal{O}_{Z\cup W} \arrow[d] \\
0 \arrow[r] & \mathcal{O}_X \arrow[r] & \cdots \arrow[r] & \mathcal{O}_X \arrow[r] & \mathcal{O}_X/I_Z \arrow[r, phantom, "="] & \mathcal{O}_Z
\end{tikzcd}$$

this map gives I_W

$\underline{Q}$. What if Z is $\mathbb{Q}$-Gorn ?