The Mathematical Education of Elementary Teachers at Delaware

Dawn Berk & James Hiebert
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Mid-Atlantic Center for Teaching and Learning Mathematics
National Science Foundation
Mathematics Preparation of Elementary/Middle School Teachers

- **3 content courses**, 1-2 methods courses, field experiences
- **Content courses** (emphasize key concepts, K-6)
  - Number and operations / place value systems
  - Rational numbers and operations
  - Geometry and algebra
- **Course improvement process (~ 7 yrs.)**
  - Study course effectiveness each semester
  - Use data to improve course before taught next time
Lesson Plans as Vehicles for Course Improvements

- Lesson plans have been developed for each course
- Course improvements are made by refining a few lessons each semester
- Lesson plan structure
  - Learning goals
  - Time-linked activities (blue font – action recommended)
  - Highlighted notes for future revisions
  - Anticipated responses by pre-service teachers and suggested instructor responses
  - Rationale for instructor based on past experience and data collected
Two Guiding Principles

• Situate pre-service teachers’ study of mathematics in contexts of teaching

• Design situations that compel pre-service teachers to confront and resolve their misconceptions
Situating Mathematics in Teaching Contexts

- Math Course 1, whole number algorithms

- Predict how children who have not yet learned the standard algorithm for multiplication might solve these problems:

  \[ 29 \times 4 = ? \]

  \[ 29 \times 12 = ? \]
Nicholas: 29 x 4 = ?
Jemea: $29 \times 12 = ?$
Helping Pre-Service Teachers Confront Their Misconceptions

• Math Course 2, writing story problems for subtraction of fractions

• Example: Write a story problem for the number sentence $\frac{5}{9} - \frac{1}{3} = ?$

• Predict the error you think pre-service teachers are most likely to make. Write a story problem of this type.
Typical Pre-Service Teacher Response

• Write a story problem for the number sentence $\frac{5}{9} - \frac{1}{3} = ?$

• Kathy has $\frac{5}{9}$ of a box of chocolates. She eats $\frac{1}{3}$ of the chocolates. How much is left?
Helping Pre-Service Teachers Confront Their Misconceptions

Problem 1
b. Kathy has 6 lbs of chocolate. She eats 1/3 of the chocolate. How much is left?
c. Kathy has 6 lbs of chocolate. She eats 1/3 lb of the chocolate. How much is left?

Problem 2
f. Kathy has ½ lb of chocolate. She eats ½ of the chocolate. How much is left?
g. Kathy has ½ lb of chocolate. She eats ½ lb of the chocolate. How much is left?
Helping Pre-Service Teachers Confront Their Misconceptions
Latest Revisions to This Lesson

• Write story problems and draw diagrams for each pair of number sentences below:

  a. $1 \frac{1}{2} - \frac{1}{3} = ?$
  b. $1 \frac{1}{2} - (\frac{1}{3} \text{ of } 1 \frac{1}{2}) = ?$
  c. $\frac{8}{9} - \frac{1}{4} = ?$
  d. $\frac{8}{9} - (\frac{1}{4} \text{ of } \frac{8}{9}) = ?$
Growth of Pre-Service Teachers’ Mathematical Knowledge for Teaching
Items Measuring Mathematical Knowledge for Teaching

- Executing procedures accurately and flexibly
- Representing quantitative situations with story problems and diagrams
- Analyzing students’ errors and nonstandard procedures
- Explaining and justifying mathematical relationships and operations
Writing Story Problems

• Write a story problem to represent the number sentence \( 2 \frac{3}{8} - \frac{2}{3} = [\ ] \)

• Write a story problem to represent the number sentence \( 1 \frac{3}{4} \div \frac{1}{2} = [\ ] \)

• Write a story problem to assess your students’ understanding of proportions
Percentage Correct (n=127)

*All changes statistically significant at the 0.05 level

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<td>22.4</td>
<td>39.8</td>
<td>75.2</td>
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Possible Explanation: “Proceduralizing the Conceptual”

- **Example**: \(2 \frac{3}{8} - \frac{2}{3} = \) [   ]

- **Correct story**: Lisa has \(2 \frac{3}{8}\) lbs of flour. She gives away \(\frac{2}{3}\) lbs. How much flour does she have left?
### Sample Result – Percent Correct

**Write a story for $1\frac{3}{4} \div \frac{1}{2} = [ ]**

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<tr>
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<th>Before Course Two</th>
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Summary

• To increase the likelihood that pre-service teachers will use their mathematical knowledge when teaching, we can situate their learning of mathematics in teaching contexts.

• Pre-service teachers’ misconceptions about key concepts can be resistant to change and require active confrontation by the pre-service teachers.