Mathematics for Elementary Teachers: A Focus on "Explaining Why"

Sybilla Beckmann

Department of Mathematics
University of Georgia

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CBMS MET Report

Teachers should have a deep understanding of the math they teach

CBMS Mathematical Education of Teachers Report, published by MAA, AMS recommends:

“Prospective teachers need mathematics courses that develop a deep understanding of the mathematics they will teach.”

Developing a deep understanding
How?

- How can college math courses help prospective elementary teachers begin to develop a deep understanding of the math they will teach?
- What features might be helpful in a text?

Features of *Mathematics for Elementary Teachers*
by Sybilla Beckmann

- A focus on "explaining why":
  - why standard procedures and formulas are valid
  - why nonstandard methods can also be valid
  - why other seemingly plausible ways of reasoning are not correct
- Centered on "Class Activities"
  - designed for use in class
  - to help prospective teachers reason about, explain, investigate, and discuss important ideas of elementary mathematics
    - often in multiple ways
    - often by drawing explicitly on fundamental principles and concepts of elementary mathematics

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Why focus on “explaining why”?

Assertions should have reasons

From NCTM **Principles and Standards for School Mathematics** (PSSM), 2000:

“From children’s earliest experiences with mathematics, it is important to help them understand that assertions should always have reasons. Questions such as “Why do you think it is true?” and “Does anyone think the answer is different, and why do you think so?” help students see that statements need to be supported or refuted by evidence.” (chapter 3)

Why focus on "explaining why"?
Reasoning is essential to understanding math

From PSSM:

"Being able to reason is essential to understanding mathematics. By developing ideas, exploring phenomena, justifying results, and using mathematical conjectures in all content areas and—with different expectations of sophistication—at all grade levels, students should see and expect that mathematics makes sense."
(chapter 3)

Teaching Mathematics in Seven Countries: *Results From the TIMSS 1999 Video Study*

Percentage of 8th grade mathematics lessons in sub-sample that contained the **development of a rationale**	
Australia	25%
Switzerland	25%
Hong Kong SAR	20%
Czech Republic	10%
Netherlands	10%
United States	**0%**

Why focus on “explaining why”?

What are the advantages?

The focus on explaining why common procedures, formulas, and solution methods of elementary mathematics are valid:

- is a means for thinking about mathematical ideas
- is a means for connecting ideas
- is a means for considering ideas from multiple perspectives and for using multiple representations
- is true to the discipline of mathematics
- is a means for focusing on fundamental concepts, principles, and ideas, such as:
  - the meanings of the operations
  - the definition of fraction
  - place value
  - properties of arithmetic
  - additivity of area and volume
  - decomposition as a generally useful technique

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What are the advantages?

Advantages of focusing on explaining why common procedures, formulas, and solution methods of elementary mathematics are valid, why nonstandard methods can also be valid, and why other seemingly plausible ways of reasoning are not correct:

- expecting to “explain why” is part of a mathematical habit of mind
- is do-able and productive
  - we can expect every teacher to be able to explain why standard solution methods and formulas are valid
  - prospective teachers believe they should know explanations
  - prospective teachers often *don’t know* that procedures and formulas *can* be explained
- provides a way to work on mathematical reasoning *within* the topics that should be the focus of elementary math instruction
- has the potential to travel into the school classroom

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The multiplication algorithm

Key ingredients in understanding and explaining it

1. Definition of multiplication: $A \times B$ means the total in A groups of B
2. Distributive property: what it says and why it should be true

$$4 \times (5+2) = 4 \times 5 + 4 \times 2$$

3. Why multiplication by 10 and powers of 10 shifts digits to the left
4. Decomposing in terms of place value and applying the distributive property multiple times

Some additional steps leading up to understanding and explaining the multiplication algorithm, shown in handout:

1. Calculating by decomposing, with percentages—done informally with pictures and “percent diagrams”
2. Showing the algebra in mental math—calculating by decomposing, now done more formally, to show the use of properties of arithmetic explicitly

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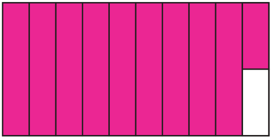
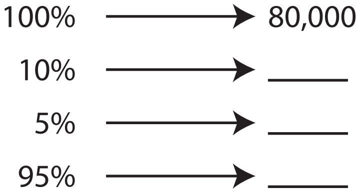
Calculating by decomposing
with benchmark fractions and percentages

Mentally calculate 95% of 80,000 by calculating $\frac{1}{10}$ of 80,000, calculating half of that result, and then taking this last amount away from 80,000.
Use the picture and the “percent diagram” to help you explain, record, and clarify your thinking.



“Percent diagram”:



Aside: a percentage problem

that can be solved with aid of a picture or percent diagram

One mouse weighs 20% more than another mouse. Together, the two mice weigh 66 grams. How much does each mouse weigh? Explain your reasoning.

For each arithmetic problem below, find ways to use properties of arithmetic to make the problem easy to do mentally. Describe your method in words, and write equations that correspond to your method. Write your equations in the form:

$$\begin{aligned} \text{original} &= \text{some expression} \\ &= \vdots \\ &= \text{some expression} \end{aligned}$$

2. 24×25. Try to find several different ways to solve this problem mentally.
6. $15\% \times \$44$

Writing equations to show the algebra in mental math
preliminary activities

A sequence of activities leading up to writing strings of equations to show the algebra in mental math:

1. Relate various basic multiplication facts via properties of arithmetic—write equations and show with pictures.

$6 \times 7 = 5 \times 7 + 1 \times 7$ $6 \times 7 = 2 \times (3 \times 7)$ $6 \times 7 = 6 \times 5 + 6 \times 2$

2. Solve multiplication problems, such as 12×125, mentally in various ways.
3. Given a string of equations that corresponds to a mental method of calculation for the previous mental multiplication problems, describe the mental method.

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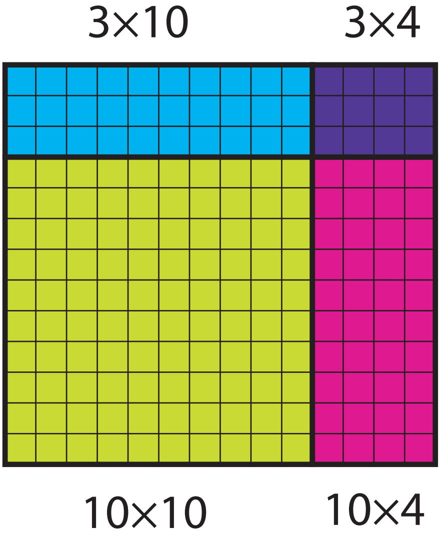
The Multiplication Algorithm

the “partial products” algorithm is a step toward the condensed standard algorithm

$$
\begin{array}{r}
14 \\
\times 13 \\
\hline
12 \\
30 \\
40 \\
100 \\
\hline
182
\end{array}
$$



Steps to understanding and explaining the division algorithm

1. Know what division means: the “how many in each group?” and “how many groups?” definitions
2. Solve division problems without using a standard procedure; analyze other people’s solution methods
3. Understand and explain the “scaffold method” of long division
4. Understand and explain the standard method of long division

Next: Activities (on the handout) that lead up to the standard division algorithm.

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Two interpretations of division

1. Use both the “how many in each group?” and the “how many groups?” interpretations of division to explain why $10 \div 2 = 5$. Write a story problem for each case. Draw simple pictures to illustrate.
2. Use both interpretations of division to explain why $14 \div 3 = 4\frac{2}{3}$. Write a story problem for each case. Draw simple pictures to illustrate.
3. Besides $4\frac{2}{3}$, what other answers could you give to the division problem $14 \div 3$? For each answer, write one or two story problems that are best answered that way. In each case, try to write one “how many in each group?” and one “how many groups?” problem.
4. Write a story problem for which you would calculate $14 \div 3$ in order to solve the problem but which has answer 5.

Dividing without using a calculator or long division

1. Zane is working on the division problem $245 \div 15$. Zane writes:

$$\begin{array}{r} 15 \\ \times 2 \\ \hline 30 \\ \times 2 \\ \hline 60 \\ \times 4 \\ \hline 240 \\ 5 \text{ left} \end{array} \qquad 2 \times 2 \times 4 = 16 \text{ R } 5$$

Explain why Zane's strategy makes sense. It may help you to work with a story problem for $245 \div 15$.
[Another part asks students to write equations in a specified form that show the use of properties of arithmetic and that correspond to Zane's work.]

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Dividing without using a calculator or long division

2. Assume that you don't have a calculator and have forgotten how to do any longhand division method. Explain how you can calculate $2783 \div 125$.

Understanding the scaffold method of long division

1. Interpret each of the steps in the scaffold below in terms of the following story problem:
   You have 3274 marbles and you want to put these marbles into bags with 8 marbles in each bag. How many bags of marbles can you make and how many marbles will be left over?

$$
\begin{array}{r}
4 \\
30 \\
400 \\
8\overline{)3475} \\
\underline{-3200} \\
275 \\
\underline{-240} \\
35 \\
\underline{-32} \\
3
\end{array}
$$

Understanding the scaffold method of long division

2. Explain how the equations

$$\begin{aligned} 3475 - 400 \times 8 - 30 \times 8 - 4 \times 8 &= 3 \\ 3475 - (400 + 30 + 4) \times 8 &= 3 \\ 3475 - 434 \times 8 &= 3 \end{aligned}$$

relate to the scaffold and to the story problem in part (1), and why the last equation shows that $3475 \div 8 = 434$, remainder 3.

Understanding the standard method of long division

Use the standard long division algorithm to calculate $1372 \div 3$. Interpret each step in the algorithm in terms of dividing 1372 toothpicks equally among 3 groups, where the toothpicks are arranged in bundles of 1 thousand, 3 hundreds, 7 tens, and 2 individual toothpicks. Show the steps pictorially by drawing how bundles will be unbundled and divided step-by-step among the 3 groups below. How is the "bringing down" step in long division related to un-bundling toothpicks?

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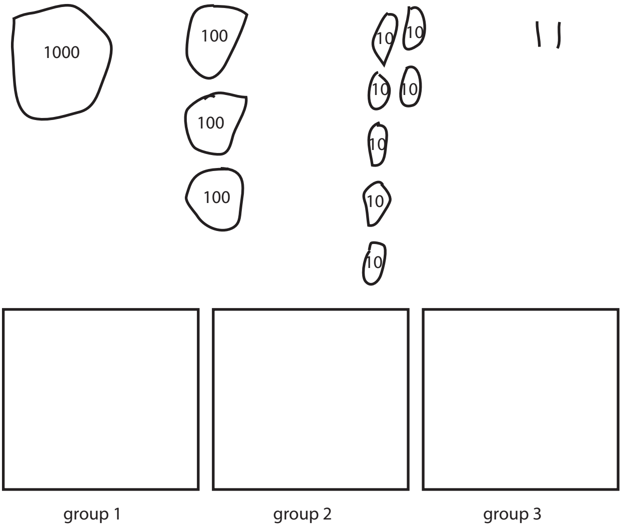
Understanding the standard method of long division



Determining area and explaining the area formula for triangles

Next set of activities on the handout:

1. Determining areas of triangles by decomposing and rearranging as well as "taking away"—an opportunity to draw explicitly on fundamental principles and to consider increasingly sophisticated methods
2. Explaining why the area formula for triangles is valid
   - first for right triangles—multiple explanations corresponding to different ways of writing the formula
   - then for non-right triangles where the height is "over the base"—multiple explanations
   - the case of oblique triangles where the height is not over the base—first consider an incorrect explanation and explain why it's not right, then explain correctly
3. Some problem solving—determining areas

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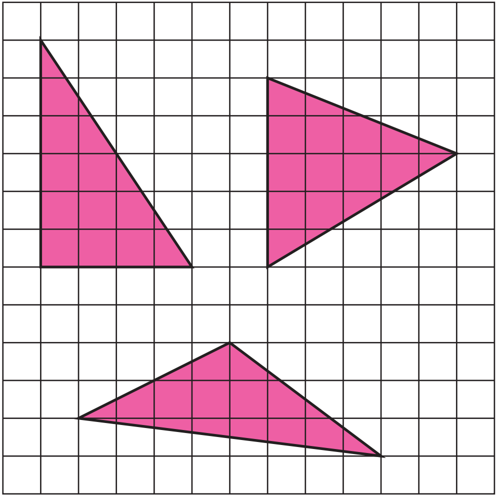
Using the moving and additivity principles

to determine areas without the triangle area formula (activity on handout)



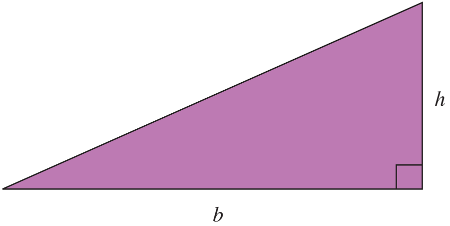
Explaining why the area formula for triangles is valid

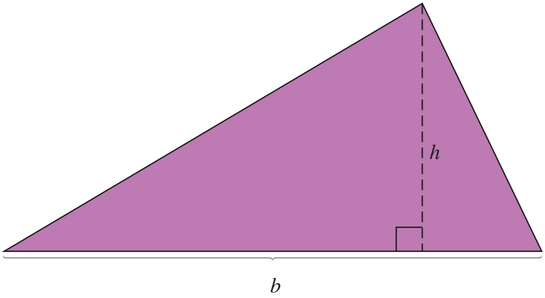
using several explanations (activity on handout)

Explain why each of the next triangles has area $\frac{1}{2}(b \times h)$ square units for the given choices of b and h.
Depending on your explanation, the formula $\frac{1}{2}(b \times h)$, the formula $(\frac{1}{2}b) \times h$, or the formula $b \times (\frac{1}{2}h)$ may be most suitable for describing the area of the triangle. Why is it valid to describe the area with any one of these three formulas?

Explaining why the area formula for triangles is valid

using several explanations (activity on handout)

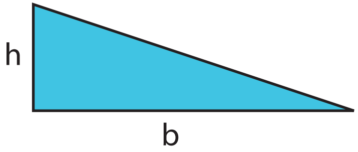




Explaining why the area formula for triangles is valid

one method, corresponding to $(\frac{1}{2}b) \cdot h$

One method:



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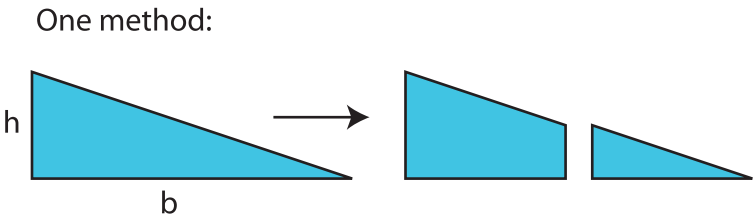

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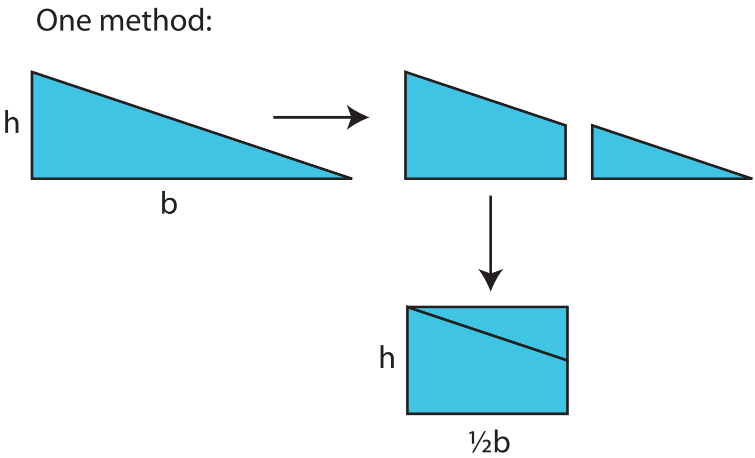

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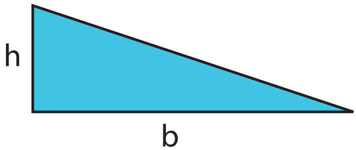
another method, corresponding to $\frac{1}{2}(b \cdot h)$

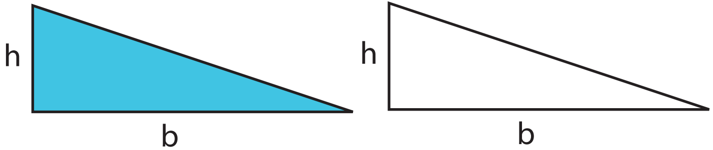
Another method:



Explaining why the area formula for triangles is valid

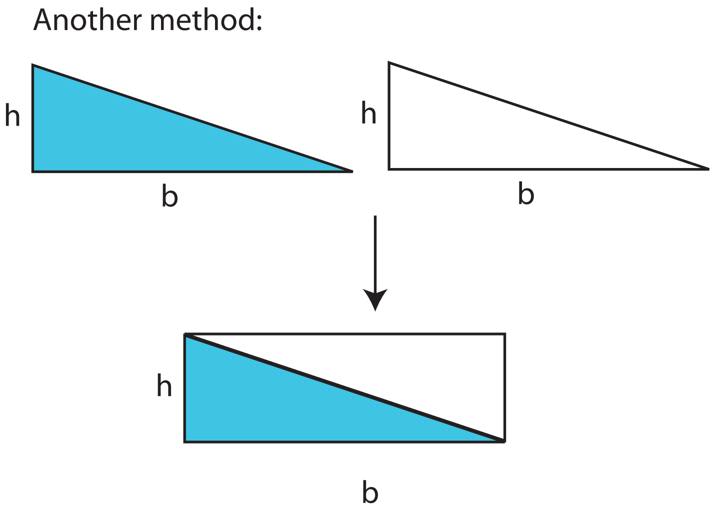
another method, corresponding to $\frac{1}{2}(b \cdot h)$

Another method:



Explaining why the area formula for triangles is valid

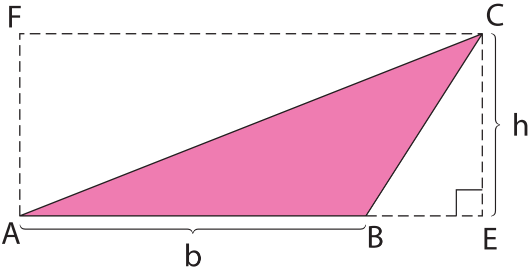
another method, corresponding to $\frac{1}{2}(b \cdot h)$



Explaining why the area formula for triangles is valid
analyze an incorrect argument in the oblique case (activity on handout)

What is wrong with the following reasoning that claims to show that the area of the triangle ABC is $\frac{1}{2}(b \times h)$ square units?

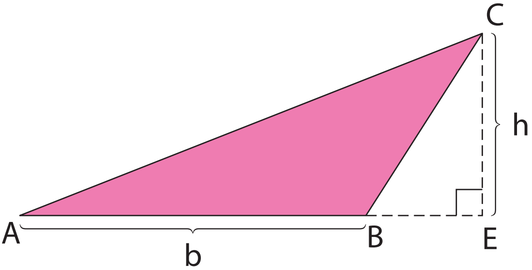
Draw a rectangle around the triangle ABC, as shown. The area of this rectangle is $b \times h$ square units. The line AC cuts the rectangle in half, so the area of the triangle ABC is half of $b \times h$ square units—in other words, $\frac{1}{2}(b \times h)$ square units.



Explaining why the area formula for triangles is valid

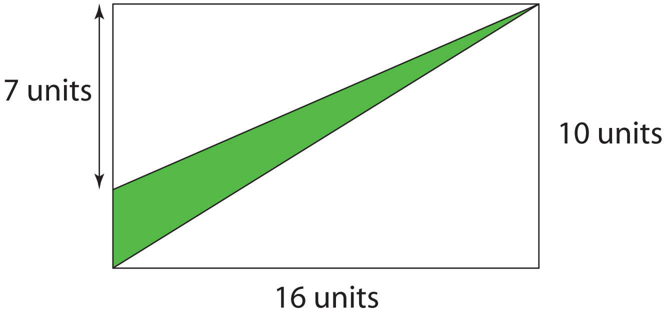
give a correct argument in the oblique case (activity on handout)

What is a valid way to explain why the triangle has area $\frac{1}{2}(b \times h)$ square units for the given choice of b and h?



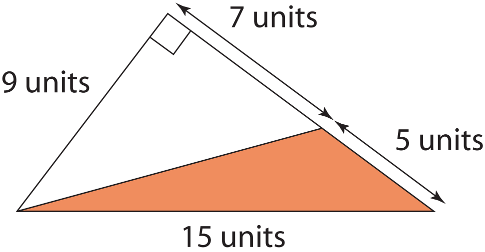
Area problem solving (activity on handout)

Determine the area of the shaded triangle that is in the rectangle in the next figure in *two different ways*. Explain your reasoning in each case.



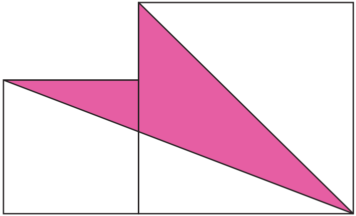
Area problem solving (activity on handout)

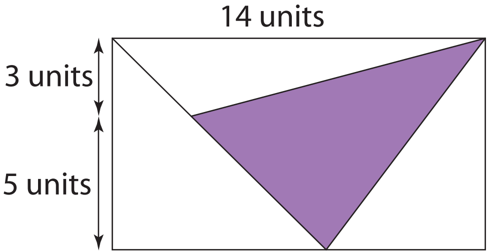
Determine the area of the shaded triangle in the next figure in *two different ways*. Explain your reasoning in each case.



Area problem solving (activity on handout)

Determine the areas of the shaded shapes in the next pair of figures. The figure on the left consists of a 3-unit by 3-unit square and a 5-unit by 5-unit square. Explain your reasoning in each case.





Explaining why seemingly plausible reasoning is not correct

a fraction comparison example

Claire says that

$$\frac{4}{9} > \frac{3}{8}$$

because

$$4 > 3 \text{ and } 9 > 8$$

Discuss whether or not Claire's reasoning is correct.

Why seemingly plausible reasoning is not correct

a fraction subtraction example

Denise says that $\frac{2}{3} - \frac{1}{2} = \frac{1}{3}$ and gives the reasoning indicated in the figure to support her answer. Is Denise right? If not, what is wrong with her reasoning and how could you help her understand her mistake and fix it? *Don't just* explain how to solve the problem correctly; explain where Denise's reasoning is flawed.