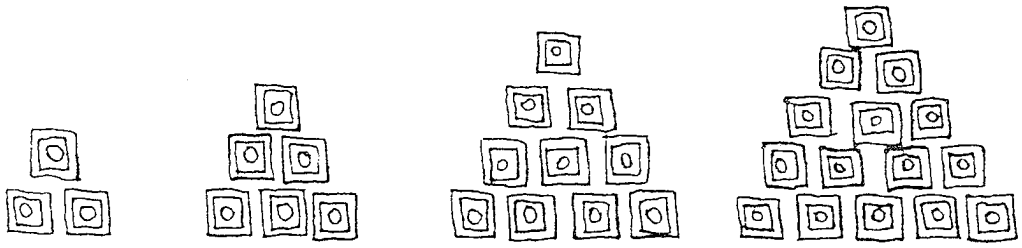
Name:_____

Challenge

CU	PS	V	Com

Given the first four steps of the pattern below:



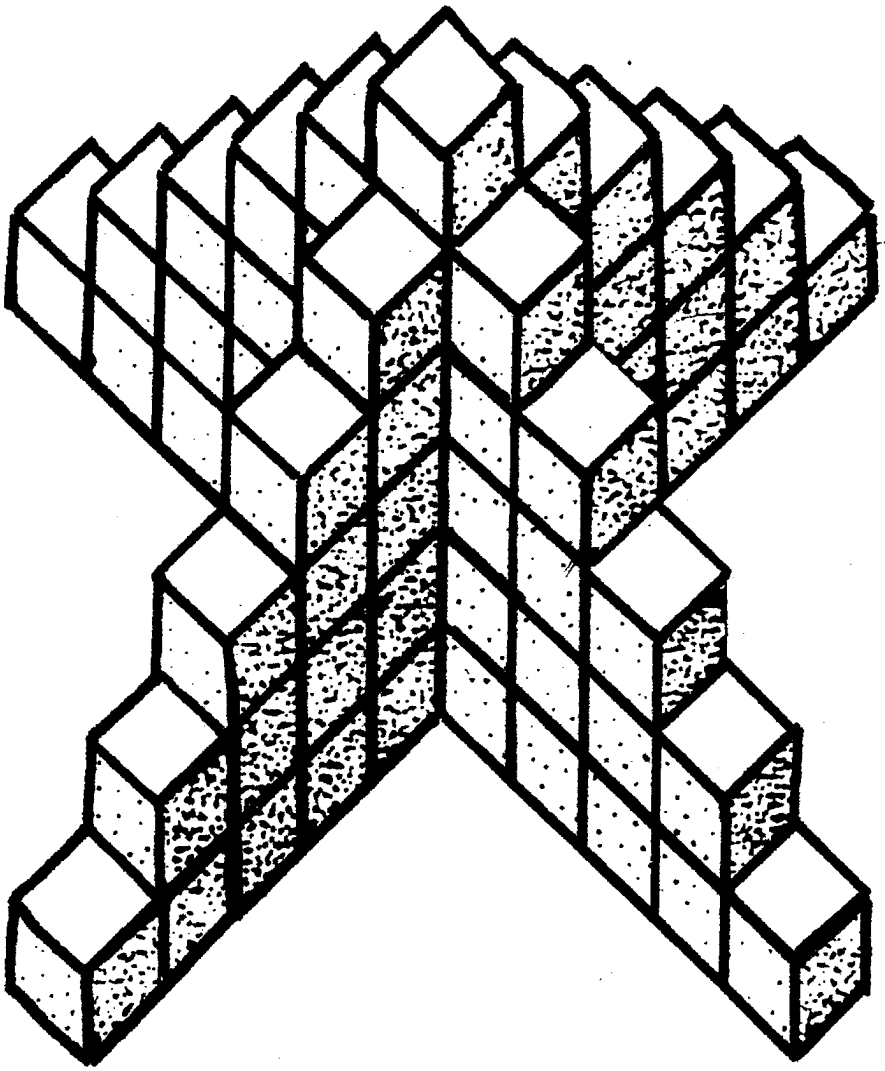
How many tiles would it take to build the 10th step?

How many tiles would it take to build the 100th step?

Challenge: How many tiles to build the 'nth' step?

Explain your reasoning.

Skeleton Tower



1. How many cubes are needed to build this tower?
2. How many cubes are needed to build a tower like this, but 12 cubes high?
3. Explain how you worked out your answer to part (2)
4. How would you calculate the number of cubes needed for a tower 'n' cubes high?
5. Graph the relationship.

Learning Together:
Implementing Inquiry-Based Content Courses for K-20 Teachers of Mathematics

Critical Issues in Education:
Teaching Teachers Mathematics Conference

Mathematical Sciences Research Institute (MSRI)
Berkeley, California
May 30 – June 1, 2007

presented by

Ruth Parker, CEO
Mathematics Education Collaborative (MEC)
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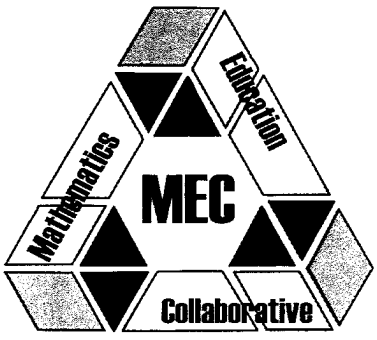


III. LESSON DESIGN AND IMPLEMENTATION

		Never Occurred				Very Descriptive
1)	The instructional strategies and activities respected students' prior knowledge and the preconceptions inherent therein.	0	1	2	3	4
2)	The lesson was designed to engage students as members of a learning community.	0	1	2	3	4
3)	In this lesson, student exploration preceded formal presentation.	0	1	2	3	4
4)	This lesson encouraged students to seek and value alternative modes of investigation or of problem solving.	0	1	2	3	4
5)	The focus and direction of the lesson was often determined by ideas originating with students.	0	1	2	3	4

IV. CONTENT

Propositional knowledge

6)	The lesson involved fundamental concepts of the subject.	0	1	2	3	4
7)	The lesson promoted strongly coherent conceptual understanding.	0	1	2	3	4
8)	The teacher had a solid grasp of the subject matter content inherent in the lesson.	0	1	2	3	4
9)	Elements of abstraction (i.e., symbolic representations, theory building) were encouraged when it was important to do so.	0	1	2	3	4
10)	Connections with other content disciplines and/or real world phenomena were explored and valued.	0	1	2	3	4

Procedural Knowledge

11)	Students used a variety of means (models, drawings, graphs, concrete materials, manipulatives, etc.) to represent phenomena.	0	1	2	3	4
12)	Students made predictions, estimations and/or hypotheses and devised means for testing them.	0	1	2	3	4
13)	Students were actively engaged in thought-provoking activity that often involved the critical assessment of procedures.	0	1	2	3	4
14)	Students were reflective about their learning.	0	1	2	3	4
15)	Intellectual rigor, constructive criticism, and the challenging of ideas were valued.	0	1	2	3	4

V. CLASSROOM CULTURE

	Communicative Interactions	Never Occurred				Very Descriptive
16)	Students were involved in the communication of their ideas to others using a variety of means and media.	0	1	2	3	4
17)	The teacher's questions triggered divergent modes of thinking.	0	1	2	3	4
18)	There was a high proportion of student talk and a significant amount of it occurred between and among students.	0	1	2	3	4
19)	Student questions and comments often determined the focus and direction of classroom discourse.	0	1	2	3	4
20)	There was a climate of respect for what others had to say.	0	1	2	3	4
	Student/Teacher Relationships					
21)	Active participation of students was encouraged and valued.	0	1	2	3	4
22)	Students were encouraged to generate conjectures, alternative solution strategies, and ways of interpreting evidence.	0	1	2	3	4
23)	In general the teacher was patient with students.	0	1	2	3	4
24)	The teacher acted as a resource person, working to support and enhance student investigations.	0	1	2	3	4
25)	The metaphor "teacher as listener" was very characteristic of this classroom.	0	1	2	3	4

Additional comments you may wish to make about this lesson.

MATHEMATICS EDUCATION COLLABORATIVE (MEC)

MEC Vision

We envision mathematics classrooms where students and teachers:

- ***Think, reason and use mathematics to make sense of the world around them***
- ***Have a sound conceptual understanding of mathematics***
- ***Use the tools of mathematics efficiently***
- ***Work persistently to solve real, challenging and relevant problems***
- ***Engage actively in learning mathematics***

MEC's mission is to work in concert with educational communities to make this vision a reality. We believe that only with a knowledgeable public – including teachers, administrators, parents and the public-at-large – will we, as a nation, secure powerful and relevant mathematics programs in our schools.

To accomplish our mission, MEC offers a series of community math nights, content courses for teachers, local leadership development, and ongoing engagement with educational leaders and business and community leaders. We view these components as essential to securing quality mathematics programs in schools.

MEC Mathematics Content Courses for K-20 Teachers of Mathematics

MEC's content courses were developed by Ruth Parker and Patricia Lofgren, and are taught by MEC instructors in school districts throughout the country. Millie Johnson, mathematics professor at Western Washington University, is developing the last three courses in the seven-course sequence. *Extending Algebraic Thinking Using a Functional Point of View* will be offered for the first time in summer of 2007 as part of the Greater Birmingham Mathematics Partnership (GBMP).

Course series:

- Patterns, Functions and Algebraic Reasoning (54 contact hours - This first course is a pre-requisite to all following courses)
- Numerical Reasoning (54 contact hours)
- Geometric and Proportional Reasoning (54 contact hours)
- Probability and Data Analysis (54 contact hours)
- Extending Algebraic Thinking Using a Functional Point of View (54 contact hours)
- Integrating Mathematics Ideas: Algebra, Geometry, Probability, and Statistics (54 contact hours)
- Connecting Mathematics Content to Science and Technology (54 contact hours)

Information about specific course content is available at MEC's website: www.mec-math.org.

Greater Birmingham Mathematics Partnership

Challenging Courses and Curricula

The National Science Foundation asked each mathematics partnership that it funds for a working definition of "challenging courses and curricula". The Greater Birmingham Mathematics Partnership (GBMP) has described the following four key aspects of challenging courses and curricula:

1. Deepening Knowledge of Important Mathematical Ideas

Deepening knowledge of important mathematical ideas includes developing a conceptual understanding of the big ideas in mathematics.

2. Productive Disposition

A productive disposition includes developing a willingness to persist in working on mathematics problems and developing confidence in one's own ability to solve mathematics problems.

3. Inquiry and Reflection

Inquiry and reflection includes using inquiry-based lessons and reflecting on learning experiences individually, in groups, and as a class.

4. Communication

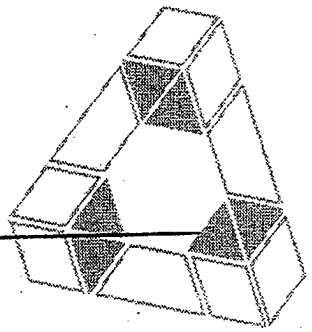
Communication includes the ability to articulate one's mathematical ideas orally and in writing to peers, teachers, parents, and others.

MEC / GBMP K-20 Partnership – Establishing Our Teaching Philosophy

The following letter was sent to the 14 university level mathematicians, engineers and mathematics and science educators at the University of Alabama at Birmingham and Birmingham Southern College prior to their participation in the first MEC course, *Patterns, Functions and Algebraic Reasoning.* It was our attempt to convey our teaching philosophy and to secure their help in interacting with the K-12 teacher participants in ways that would support course goals.

Mathematics Education Collaborative

PARTNERS IN SUPPORT OF MATHEMATICS EDUCATION



May 31, 2005

Dear IHE Faculty Members,

We at MEC are excited that you will be joining us for the Patterns, Functions and Algebraic Reasoning course this summer. We think it will provide an interesting opportunity to observe teacher learning, and we also hope that the experience will provoke thoughtful conversations about mathematics teaching and learning. Typically, K-12 teachers participate together in this course, and this will be the first time that substantial numbers of university faculty will be involved. With this in mind, we would like to offer some things for you to think about ahead of time. You may or may not agree with the ideas expressed below, but it is important that you know some of the principles that guide our decision-making in the course.

First, we view confusion - the cognitive conflict that accompanies 'not knowing' - as a natural and even desirable part of the process of constructing new understandings. We work hard to ensure that course participants have opportunities to struggle with problems, to find their own ways of solving them, and to recognize that there is usually not just one way to solve a mathematics problem. In their struggle to make sense of the problems we pose, many teachers encounter old math fears or phobias in this course, and often even those who are not fearful still struggle with being in a state of 'not knowing' as learners. We have purposefully designed the course to provide opportunities for participants to confront and get beyond their initial discomfort. The dilemma for instructors is that we were taught that a teacher's job is to help or teach by giving clear explanations of how to best solve problems. We have learned, however, that this natural inclination to want to put confusion to rest, and to 'help' those who are struggling is often counterproductive to developing mathematical understandings.

One idea that permeates our work is the belief that all learners are capable of mathematical thinking and of 'having powerful mathematical ideas'. We know that for this to occur, learners must be given adequate time to grapple with and make sense of problems in their own ways. This requires that we, as teachers, resist our natural tendencies to give explanations or help in ways that put confusion to rest prematurely. As instructors, this continues to be an ongoing challenge for us, especially early in the course when teachers actively seek out our guidance and want explanations from us. To promote teachers as active and able mathematical problem solvers we, as instructors, strive to resist giving explanations or directions toward a solution. Although we will often ask a clarifying question, we try to convey by our words and actions that we believe all students are capable of making sense of the mathematics they encounter.

Just a quick word about our use of the word confusion… We know that the word means different things to different people, and we don't want to leave the impression that we view all confusion as desirable. Some kinds of confusion need to be 'cleared up' (e.g., when some kind of 'social knowledge' such as how a symbol is used or how a problem is posed has not been made clear). One aspect of the course that you will want to pay attention to is how course instructors interact with participants in large group, small group and individual settings. Observe and think about the actions instructors take, and watch the impact of those actions on learners over the nine days. Notice what is done to meet the range of learner needs. Observe how confusions do (or do not) get resolved over time. We hope to have your feedback, and we would welcome your ideas and suggestions on this and any other aspect of the course.

We anticipate that course participants will look to you for your expertise when they are struggling with a mathematical idea or trying to solve a problem. We ask that you resist taking a lead in solving problems when you are working with groups or individuals, and resist offering explanations that may help learners out of their cognitive conflict. This will require a special awareness and effort on your part. We do want you to be engaged as learners during the course, and we hope you will find some of the mathematics tasks interesting and challenging. We also hope you will refrain from sharing your expertise in ways that direct how a group or individual approaches any given task. When participants turn to your for help, as they certainly will, we hope that you will interact with them in ways that don't direct their thinking. Rather than helping solve a problem for a group or individual, we hope you will use the opportunity to study the situation and really observe teachers as learners of mathematics. Let teachers find their own ways through the problems, and honor their struggles. As instructors, we have come to believe that 'teaching by telling' rarely leads to the deep mathematical understandings or productive mathematical dispositions we hope to promote.

We do not plan to single out the IHE faculty members by introducing you at the beginning of the course and explicitly explaining what your role entails. You will be assigned randomly to groups along with all the other participants. Several IHE faculty members who have participated in the course in the past have enjoyed this process. They joined the groups and engaged with the problems but did a lot more thinking than talking.

They contributed in small appropriate ways but tried hard not to interfere with or lead the thinking of others. They did not remain aloof nor did they tell others that they were there to watch rather than help. We invite you to experience the course in a similar fashion, and we trust it will be a rich learning opportunity for you.

A guiding principle that permeates all of our coursework is captured in the following quote:

> *I used to think that my job was to teach children to see what I see. I don't believe that anymore. I now understand that my job is to teach children to see, and recognize that we don't all see things the same way. I want my students to know that when we examine our different ways of seeing, and look for relationships among the ideas, deeper mathematical relationships are often revealed, and we ALL deepen our understanding of the mathematics involved. We all see more clearly as a result of having worked to understand our different perspectives.*
>
> *Ruth Parker, MEC*

As participants in the course, teachers experience and come to see the value of these ideas. We hope you will as well, and we look forward to working with you as we move forward with the Greater Birmingham Mathematics Partnership.

Best Regards,

Ruth Parker and Patty Lofgren
Mathematics Education Collaborative (MEC)

Teacher Growth On GBMP Characteristics of Challenging Courses (CCC) in MEC's 9-day Patterns, Functions and Algebraic Reasoning Course

Three randomly selected teachers were each shadowed by two evaluators on day 1, Day 4, and Day 8 of the course.

Behavioral Checklist Results for Selected Participants by CCC Dimension

	Participant 1			Participant 2			Participant 3		
	Day 1	Day 4	Day 8	Day 1	Day 4	Day 8	Day 1	Day 4	Day 8
Understanding of Mathematical Ideas									
Uses variables to describe unknowns		X	X			X	X	X	X
Explains why equations make sense geometrically			X			X		X	X
Represents linear and quadratic equations in variety of ways			X			X		X	X
Productive Disposition									
Persists when answer is not known		X	X			X		X	X
Asks for guidance but not answers			X			X		X	X
Tries variety of strategies for approaching problem			X			X			X
Inquiry and Reflection									
Makes extensions and connections beyond immediate problem			X					X	X
Explores why it works and whether it will always work									X
Confusion and mistakes lead to further exploration			X			X			X
Communication									
Explains reasoning fluently			X			X			X
Asks probing questions			X						X
Shares ideas with class		X	X			X		X	X

RTOP Classroom Observation Data

Year 1 is baseline data – Classroom observations were done prior to first MEC course
Year 2 is after MEC's 9-day course, Patterns, Functions and Algebraic Reasoning
Year 3 teachers have each had one or two MEC courses
Random Sample of Teachers, each observed by two trained evaluators

Year 1 (n=11)				Year 2 (n=12)				Year 3 (n=40)			
G5	G6	G7	G8	G5	G6	G7	G8	G5	G6	G7	G8
1	1	4	5	1	1	7	3	10	9	9	12
Bessemer Fairfield Hoover Homewood Jefferson County Shelby County				Fairfield Hoover Homewood Jefferson County Shelby County Trussville				Bessemer Fairfield Hoover Homewood Jefferson County Mountain Brook Shelby County Trussville			

Each of the categories below consists of five items. Items are rated on a five-point scale ranging from 0 to 4. Observers select a "0" if the characteristic never occurred in the lesson to a "4" if the item was very descriptive of the lesson.

Median Ratings by Category

RTOP Categories	Year	Median
Lesson Design/Implementation	1	4
	2	11.75
	3	14.5
Propositional Knowledge	1	5.5
	2	10.75
	3	13.25
Procedural Knowledge	1	4
	2	13
	3	13.5
Communicative Interaction	1	4
	2	12.75
	3	13.5
Student/Teacher Relationships	1	5.5
	2	14
	3	15.5

For Your Algebra Portfolio

You will be selecting pieces that include the following:

- A reflective piece on:
  - you as a learner of mathematics
  - you as a teacher of mathematics;
- A letter to a colleague or administrator or to the ESD that synthesizes your "big ideas" from the course and that describes the kinds of support that you will need in order to move forward in restructuring your mathematics teaching;
- Your most important piece of work from the course;
- A scored task from the course;
- A selection of your choice that you think helps complete the picture of you as a user/teacher of mathematics;
- An assigned assessment task from the course.

Grading Criteria

Criteria for an A

1. Oral and written communications are clear, convincing and mathematically sound;
2. Demonstrates evidence of persistence when solving problems;
3. Completes all requirements of the course including full participation, homework, assessment tasks, and portfolio.
4. Able to explain why equations make sense geometrically;
5. Makes connections within a problem (its various representations) and to other problems;
6. Demonstrates procedural fluency with real number operations and symbolic manipulations;
7. Consistently demonstrates ability to accurately represent linear and quadratic relationships using and making connections among multiple representations (i.e. equations, tables, graphs, verbal/situational, geometric/visual); and,
8. Frequently extends thinking beyond the immediate problem.

Criteria for a B

1 through 6 above, plus:

7. Consistently represents linear relationships using a variety of representations (e.g. expressions or equations, tables, graphs, etc.);
8. Generally represents quadratic relationships using a variety of accurate representations (e.g. equations, tables, graphs, etc.); and,
9. Sometimes extends thinking beyond the immediate problem.

Criteria for a C

1 through 4 above, plus:

5. Makes connections within a problem and its various representations; and,
6. Demonstrates ability to accurately represent linear relationships using a variety of representations (e.g. equations, tables, graphs, etc.).

**University credit requires full attendance.

Results of Course Evaluation Survey
Summer 2005

ITEMS	Participant Group	# of Respondents	Strongly Agree	Agree	Disagree	Strongly Disagree
1. This course improved my mathematical skills and understanding.	Gr. K-4	44	40 *91%*	4 *9%*	0 *0%*	0 *0%*
	Gr. 5-8	93	81 *87%*	10 *11%*	2 *2%*	0 *0%*
	Gr. 9-12	8	5 *63%*	3 *37%*	0 *0%*	0 *0%*
	Pre-service	9	9 *100%*	0 *0%*	0 *0%*	0% *0*
	IHE Faculty	9	5 *56%*	3 *33%*	1 *11%*	0 *0%*
	Other	8	7 *88%*	1 *12%*	0 *0%*	0 *0%*
	ALL	**171**	**147** ***86%***	**21** ***12%***	**3** ***2%***	**0** ***0%***
2. The mathematical ideas presented in this course will be useful in my teaching during the upcoming school year.	Gr. K-4	44	32 *73%*	11 *25%*	1 *2%*	0 *0%*
	Gr. 5-8	92	79 *86%*	11 *12%*	2 *2%*	0 *0%*
	Gr. 9-12	8	5 *63%*	3 *37%*	0 *0%*	0 *0%*
	Pre-service	8	8 *100%*	0 *0%*	0 *0%*	0 *0%*
	IHE Faculty	9	4 *44%*	4 *44%*	1 *12%*	0 *0%*
	Other	8	6 *75%*	2 *25%*	0 *0%*	0 *0%*
	ALL	**169**	**134** ***79%***	**31** ***18%***	**4** ***3%***	**0** ***0%***
3. The teaching modeled in this course will be useful to me during the upcoming school year.	Gr. K-4	43	36 *84%*	6 *14%*	1 *2%*	0 *0%*
	Gr. 5-8	93	77 *83%*	15 *16%*	1 *1%*	0 *0%*
	Gr. 9-12	8	4 *50%*	4 *50%*	0 *0%*	0 *0%*
	Pre-service	8	6 *75%*	2 *25%*	0 *0%*	0 *0%*
	IHE Faculty	9	6 *67%*	3 *33%*	0 *0%*	0 *0%*
	Other	8	6 *75%*	1 *13%*	1 *13%*	0 *0%*
	ALL	**169**	**135** ***80%***	**31** ***18%***	**3** ***2%***	**0** ***0%***

ITEMS	Participant Group	# of Respondents	Strongly Agree	Agree	Disagree	Strongly Disagree
4. This course improved my understanding of pedagogy/teaching practices.	Gr. K-4	44	35 *80%*	8 *18%*	1 *2%*	0 *0%*
	Gr. 5-8	93	71 *76%*	21 *23%*	1 *1%*	0 *0%*
	Gr. 9-12	8	3 *38%*	5 *62%*	0 *0%*	0 *0%*
	Pre-service	9	7 *78%*	2 *22%*	0 *0%*	0 *0%*
	IHE Faculty	9	7 *78%*	2 *22%*	0 *0%*	0 *0%*
	Other	8	5 *62%*	3 *38%*	0 *0%*	0 *0%*
	ALL	**171**	**128** ***75%***	**41** ***24%***	**2** ***1%***	**0** ***0%***
5. The materials in the kit provided will be useful in my teaching during the upcoming school year.	Gr. K-4	---	NA	NA	NA	NA
	Gr. 5-8	93	76 *82%*	16 *17%*	1 *1%*	0 *0%*
	Gr. 9-12	7	4 *57%*	3 *43%*	0 *0%*	0 *0%*
	Pre-service	---	NA	NA	NA	NA
	IHE Faculty	---	NA	NA	NA	NA
	Other	---	NA	NA	NA	NA
	ALL	**101**	**80** ***79%***	**19** ***19%***	**1** ***1%***	**0** ***0%***
6. I found the course interesting and engaging.	Gr. K-4	44	39 *89%*	4 *9%*	1 *2%*	0 *0%*
	Gr. 5-8	93	82 *88%*	9 *10%*	2 *2%*	0 *0%*
	Gr. 9-12	8	4 *50%*	4 *50%*	0 *0%*	0 *0%*
	Pre-service	9	9 *100%*	0 *0%*	0 *0%*	0 *0%*
	IHE Faculty	9	8 *89%*	1 *11%*	0 *0%*	0 *0%*
	Other	8	6 *75%*	2 *25%*	0 *0%*	0 *0%*
	ALL	**171**	**148** ***87%***	**20** ***12%***	**3** ***1%***	**0** ***0%***
7. The instructor was knowledgeable and effective.	Gr. K-4	44	44 *100%*	0 *0%*	0 *0%*	0 *0%*
	Gr. 5-8	93	91 *98%*	2 *2%*	0 *0%*	0 *0%*
	Gr. 9-12	8	7 *88%*	1 *12%*	0 *0%*	0 *0%*
	Pre-service	9	9 *100%*	0 *0%*	0 *0%*	0 *0%*
	IHE Faculty	9	8 *89%*	1 *11%*	0 *0%*	0 *0%*
	Other	8	7 *88%*	1 *12%*	0 *0%*	0 *0%*
	ALL	**171**	**166** ***97%***	**5** ***3%***	**0** ***0%***	**0** ***0%***

From: Golliher20@aol.com
Date: August 4, 2006 6:21:30 AM PDT
To: ruthp@mec-math.org, pattyl@mec-math.org
Subject: (no subject)

I wanted you to know how well our school did on the state tests. I know that the inappropriate use of testing is one of your concerns, but overall, I think CSAP is a reasonable test and it has made teachers and schools focus on math, which is not a bad thing. Our students do number talks daily and teachers use Investigations as the core program. In Title 1, we use Kathy Richardson's assessment system in grade 2-4 to guide instruction and then use Knowing Mathematics with our Title 1 kids in 5th grade. This year, Michelle Faye moved to Rocky Mountain Elementary to work as a .5 math coach, so she helped the teachers while I continued to work with the Title 1 kids.

The results of this year's tests were release yesterday and we received an award from the Governor because we made more improvement than any other school in the state. For a school with more than 75% of its students on free or reduced lunches and well over 60% of the students bilingual, we did exceptionally well. In 3rd and 4th grades, NO students scored in the "unsatisfactory" range. In fact, in 3rd-5th grades we only had 2 students score unsatisfactory. Remember, a large percentage of our students are native Spanish speakers and the test is in English. In third grade, we can translate the test orally so they are hearing it in Spanish but looking at the words in English which I think would be very confusing.

In 2003, 18% of the 5th graders at our school were Proficient or Advanced, this year 63% were in those categories. In third grade 76% were proficient or advanced--in fact, we beat the district and state scores for 3rd grade. Michelle was interviewed by the papers and TV and she did a fabulous job. In the only part of the interview that they showed on TV, she said, "We used to teach kids to memorize, but now we teach them to think." In the interviews in the paper she emphasized the need to link assessment to instruction and meet as grade level teams to analyze the data and plan instruction.

So, all our work with the MST really paid off. Our kids are learning a lot more math and we are showing that good teaching makes the difference even if your students come from challenging backgrounds. It is a good feeling to leave teaching knowing that our work together did make a difference.

Please take care of yourselves and stay safe!

Mark St. John, On Why Working with Parents and the Public Matters

From: "MARK ST JOHN" <Mstjohn@inverness-research.org>
Date: December 9, 2004 6:40:27 PM PST
To: "'Ruth Parker'" <ruthp11@earthlink.net>

Ruth...

The basic idea, as I understand it, has to do with what I call internal and external validity. That is, any new math program certainly has to have internal validity...be of high quality, well designed, efficacious in helping students learn etc... But it also has to have external or contextual validity... Ten or so years ago external validity was not so important perhaps. Now it is the major issue. The program might be good but if it does not address the values and demands and perceptions of the community context, it is dead in the water.

Hence there is a need to understand the context... and not superficially... and then to use that understanding to begin to create... really to engineer...a supportive climate. This involves a two way flow of understanding... parents need to understand the math program...and the math advocates need to understand and address the very legitimate concerns of the parents....

So I see you doing important work in trying to figure out how to engineer a supportive context for math improvement... and how to help math educators better speak to that context...

The idea of doing this work, and then studying it speaks to me of important design research... That is, an integrated approach of helping many projects design and carry out engineering efforts, and then studying those efforts to garner more general lessons learned....

Mark St. John
Inverness Research Associates
Box 303
Inverness, CA 94937
415-669-7156
415-669-7186 (fax)

To Ruth Parker from Mark St. John – 8/11/06

Ruth… there are two aspects to the work of improving mathematics instruction. One is in a sense substantive and technical. That is, one aspect involves the design and implementation of better instruction so that students have a better opportunity to learn mathematics. This work involves creating better curriculum and better teacher supports. Also better assessments.

The other aspect of improving math instruction involves the creation of a supportive context for the improvement effort. That is, even if we know how to improve instruction, it will not happen unless there is a context that is willing to support the work, and even create demand for the work. Or put the other way, there is no hope of improvement if the context is resistant and unwilling and unconvinced. Here the status quo, no matter how horrible, has a real advantage.

The supportive context involves policies, resources, and ultimately opinions.

In my own study of reform efforts, the barrier to improvement has shifted from substantive to context. Now the key problem to solve is the problem of engineering a supportive context. The technical and substantive challenges are still there, but they are not the limiting factor. It is the lack of public understanding and support that is the bottleneck to improvement. Hence, your work is key.

See our website for a study of high school curriculum implementation to see how the context is what ultimately kills the improvement effort every time.... www.inverness-research.org. \
Best. Mark

TEACHING FOR UNDERSTANDING: GUIDING PRINCIPLES*

The type of mathematics instruction that involves students actively and intellectually requires much from the teacher. Without thoughtful decisions about the particular activities and without thoughtful interactions with students, potentially powerful mathematical experiences can become little more than interesting activities for students. The following basic principles are important to keep in mind when implementing a mathematics program that gives high priority to the development of understanding.

1. Our top priority should be the development of students' thinking and understanding. Whenever possible, we should engage the students' thinking and teach the mathematical ideas through posing a problem, setting up a situation, or asking a question.

2. We must know that understanding is achieved through direct, personal experiences. Students need to verify their thinking for themselves rather than to depend on an outside authority to tell them if they are right or wrong. We must see our job as setting up appropriate situations, asking questions, listening to children, and focusing the attention of the students on important elements rather than trying to teach a concept through explanations.

3. We must know that the understandings we seek to help the students gain are developed, elaborated, deepened, and made more complete over time. We must provide a variety of opportunities to explore and confront any mathematical idea many times.

4. We will not expect all students to get the same thing out of the same experience. What students learn from any particular activity depends in large part on their past experiences and cognitive maturity. We should try to provide activities that have the potential for being understood at many different levels.

5. To maximize the opportunities for meaningful learning, we should encourage students to work together in small groups. Students learn not only from adults but also from each other as well.

6. We must recognize that partially grasped ideas and periods of confusion are a natural part of the process of developing understanding. When a student does not reach the anticipated conclusion, we must resist giving an explanation and try to ask a question or pose a new problem that will give the student the opportunity to contemplate evidence not previously considered.

*California Mathematics Model Curriculum Guide

7. We must be interested in what students are really thinking and understanding. Students may be able to answer correctly but still have fundamental misunderstandings. It is through the probing of the students' thinking that we get the information we need to provide appropriate learning experiences.

8. We must be clear about the particular idea or concept we wish students to consider when we present activities or use concrete models. It is not the activities or the models by themselves that are important. What is important is the students' thinking about and reflection on those particular ideas dealt with in the activities or represented by the models.

9. We need to recognize that students' thinking can often be stimulated by questions, whether directed by the teacher or other students. We should foster a questioning attitude in our students.

10. We need to help students develop persistence in solving problems. Only in a learning environment in which mistakes and confusion are considered to be a natural part of the learning process can students believe they do not have to come up with quick, right answers.

11. We need to recognize the importance of verbalization. Putting thoughts into words requires students to organize their thinking and to confront their incomplete understanding. Listening to others affords them the opportunity to contemplate the thinking of others and to consider the implications for their own understanding.

12. We must value the development of mathematical language. Language should serve to internalize and clarify thinking and to communicate ideas and not be an end in itself. Memorizing definitions without understanding interferes with thinking. The emphasis is on developing a concept first, establishing the need for precise language, and then labeling the concept accurately.