Learning and Teaching Linear Functions

Video Cases for Mathematics Professional Development, 6-10

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Heinemann

Overview

These video-based mathematics professional development resources were created to help teachers' address some issues and challenges specific to teaching linear functions. The materials, based on videos of real classroom teaching, are designed to better equip teachers to prepare and enact lessons that will help students develop conceptual understanding of linear functions. It is intended that teachers' own understanding of linear functions be deepened while focusing on the mathematics within teaching.

Materials consist of one foundation module consisting of eight three-hour sessions, and four extension modules, each consisting of two or three three-hour sessions. Each module has been constructed as a sequence of interrelated cases built around learning goals designed for use with mathematics teachers in grades 6-10, with a mathematical focus on algebra—specifically linear functions. Each case (designed for a 3-hour session) consists of mathematics tasks, one or two short 3-8 minute video clips with discussion tasks, readings, and tasks designed as a bridge to teachers' practice. The video segments are unedited clips from lessons filmed in actual mathematics classrooms (grades 3-10). Materials utilize CD-ROM technology employing the LessonLab, Inc. format for video. Facilitator Guide, agendas and resource materials are in a printable PDF format on the CD. Sample PowerPoint slides are also included on the CD. Resource materials consist of transcripts, lesson graphs, mathematical commentaries, and related reading materials.

Overarching Goals of the VMCPD Materials

The modules are designed to provide teachers with opportunities to study and analyze the complex and subtle nature of teaching mathematics. The goal of this professional development experience is to help teachers develop the knowledge, skills and sensibilities to reason and make informed decisions about their own teaching. Experiences are designed around some tasks of teaching mathematics during whole class settings:

- Keeping one's eye on the mathematical trajectory (which means knowing the mathematics and one's goals for students' learning mathematics);
- Choosing, using and comparing various representations of mathematics to further students' learning (mathematical representations as well as student ideas and approaches);
- Using student productions as opportunities for all students' learning.
- Interpreting and responding to perceived student errors and unexpected student methods

Overview of Modules

These materials are comprised of one foundation and four extension modules for a total of 54 hours of professional development. The thread of teaching in the context of linear relationships ties all of the modules together. The foundation module focuses deeply on the mathematics of representing and conceptualizing linear relationships, with the extension modules building from that foundation. The five modules are briefly described below:

Research

The foundation module, *Teaching Mathematics: Conceptualizing and Representing Linear Relationships*, was piloted in five different sites across the country. Two separate evaluation efforts assessed various aspects of teacher learning from these materials. Heather Hill and Rachel Collopy developed and used an external assessment to measure growth in teachers' content knowledge and pedagogical content knowledge. They conducted pre- and post-workshop administrations of this assessment with a group of twelve LTLF teachers and a comparison sample of ten teachers. LTLF teachers improved in their ability to: algebraically represent problems involving geometric patterns, connect their algebraic representation to the geometric pattern, and compare and link alternative representations of the same linear function. They also were better able to identify potential student misunderstandings that involved using a recursive method for predicting the next term in a sequence. Given the small sample size, Hill and Collopy did not ascertain the extent to which teachers' growth between the pre- and post-tests was statistically significant. However, equivalent growth did not appear in the comparison group, which provides some assurance that the improvement did not result from retaking the same items over a relatively short time span (Hill and Collopy, 2002).

Horizon Research, under the guidance of Iris Weiss and Dan Heck, developed embedded assessments to measure the impact of the LTLF materials on teachers' content knowledge and pedagogical content knowledge. Embedded means that the instruments are connected to the actual work teachers do within sessions. In this case, the embedded assessment instrument administered at the start of the foundation module required teachers to solve a mathematics task, reflect on their approach to solving the task, and predict approaches students might use to solve it. Near the end of the module they were asked to complete a similar process with another mathematics task. The second embedded assessment involved a pre- and post-video analysis task. A comparison group of similar teachers responded to the same tasks. In terms of exhibiting mathematical knowledge pertinent to the teaching of mathematics, LTLF participants were statistically more likely than control group teachers to increase in their ability/propensity to connect their work on the mathematics tasks back to the pictorial representations in which the task originated (Heck, 2003).

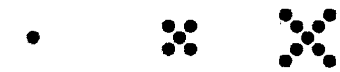
Turning to the Evidence (TTE) (REC- 0231892; 2002-present; Goldsmith & Seago, co-PIs) examined the impact of two PD programs focused on algebraic thinking: *Learning and Teaching Linear Functions: VideoCases for Mathematics Professional Development* (*LTLF*) (Seago et al., 2004) and *Fostering Algebraic Thinking Toolkit* (*AT*) (Driscoll et al., 2001). Four PD seminars were conducted for this project, two *AT* groups (facilitated by Driscoll) and two *LTLF* seminars (facilitated by Seago). Seventy-four U.S. middle and high school teachers participated in this study: 49 as PD participants and 25 as a comparison group. Sixteen case study teachers (four from each site) were followed more closely. Results indicate teachers across both sets of materials learned to be more analytic about student thinking: performance on post-program written assessments indicated that participants' analyses of video and written student work were more grounded in evidence, more focused on the mathematics captured in the artifacts, and more attentive to the mathematical potential of students' ideas (instead of just the correctness of the work) than those of a comparison group (Goldsmith et al., 2005). Analysis of seminar discourse indicates participants' analysis of mathematical thinking became more sustained, extensive, and nuanced over time, and they developed more differentiated, representation-rich, and flexible approaches to the mathematics (Goldsmith et al., 2005; Seago & Goldsmith, 2006).

[50 minute lesson]

Session One: Conceptualizing and Representing Linear Relationships
Lesson Graph: Growing Dots 1 Lesson, Period 4 [HS Algebra]

4 minutes

Posing the problem Kirk puts one poster up at a time on the white board.


At the beginning — At 1 minute — At 2 minutes

Kirk posts the task on the board:
Describe the pattern. Assuming the sequence continues in the same way, how many dots are there at 100 minutes? Create a table and graph. Write an equation for the number of dots at t minutes.

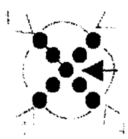
7 minutes

Students work on problem individually or with partners
Students choose others to work with, some move their desk. Kirk circulates as students work on the problem.

16 minutes

Whole-class sharing of solutions Kirk asks students to share their solutions:

$x \cdot 4 + 1$ **Danielle**



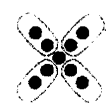
The center or 1 in the equation. 4 would be all the dots except in the center. x is how many dots out from the center.

James $x + 4$

x is the previous picture. That plus 4 is the next picture.

1 + 4 = 5

Markiesha $4x + 1$



The first minute there was 1 dot on the outside corners, the 2[nd] minute had 2 dots, at 100 there would be 400 dots, 100 on each plus one in the middle.

Kirk notes that two methods came up with 401 for 100 dots and one has 400. He asks James if he is still sticking with his solution. James says yes, because the plus 1 doesn't make sense to him. Kirk clarifies what James' x means—previous # of dots and what Danielle and Markiesha's x means—minutes.

Working backward Kirk asks the class: at how many minutes will you have 25 dots? 73 dots? 99 dots? Students spend a few minutes discussing this. Then Kirk asks for students to share their solutions and methods for 25 dots:

Marcella
$25 = 4t + 1$
$-1 \quad -1$
$\frac{24}{4} = \frac{4t}{4}$
$6 = t$

James
counted—kept adding four each time. He said Marcella's way is easier.

Janelle

0	1
1	5
2	9

I continued on, adding 4 to 5 min and got 25 dots.

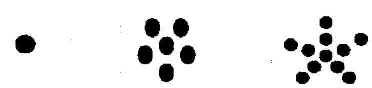
John says he did the opposite: → When Kirk asks him what he means by opposite, he says he did the opposite of the equation $4x + 1$: subtract 1 and divide by 4.
I subtracted 1 from 25, got 24 and divided by 4. He shows:
$\frac{x - 1}{4}$

Students share that they got 18 for 73 and 24.5 for 99. James says he got 97 and then just said half. Kirk asks what the picture will look like and students respond half of a dot all the way around. Kirk says we don't know how its growing, only what it looks like at each minute. A student replies: we are assuming it goes in the x pattern.

9 minutes

Posing of additional growing dot problem Kirk says: I have one more dot pattern for you.

Kirk posts the task on the board asking students to use their same graphs/papers:
Describe the pattern. Assuming the sequence continues in the same way, how many dots are there at 100 minutes? Create a table and graph. Write an equation for the number of dots at t minutes.

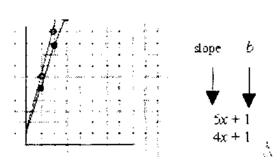

At the beginning — At 1 minute — At 2 minutes

Students work on problem James asks Kirk, isn't this the same as the other one? If Danielle's equation is right then her equation is the same as this $4t + 1$. Kirk asks if in the new problem are we going by 4? James responds it's going by 5.

10 minutes

Whole-class discussion of 2[nd] problem
Janelle's group shares that they got 501 for 100 dots. Kirk puts up Janelle's graph from the 1[st] dot problem. Danielle volunteers to graph the 2[nd] dot pattern in blue on the overhead.

Kirk says: Compare the two graphs. How are they similar? How are they different?
Students say:
Dots on the same line, but in a different place. It is like it stretched, the slope is getting steeper. Kirk asks why is the slope getting steeper? Students respond more dots; more to measure. You always started with one at the beginning of the graph. A student says that is where the y-intercept is. James says that the m is different and the b is the same.



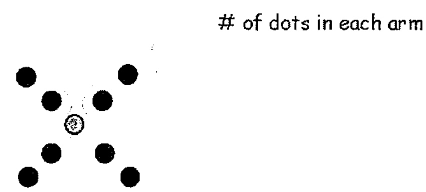
4 minutes

Extension of the problem
Kirk asks: What would the equation be for a dot pattern with 3 arms? 100 arms? n arms? Think about that for a minute. A student responds she did the same thing as she did for 4 except she was adding 3 and got: $3t + 1$. For 100 arms, students respond $100t + 1$ and $nt + 1$ for n arms. Kirk asks what n represents and students respond number of arms. He asks what the t represents and they respond time.

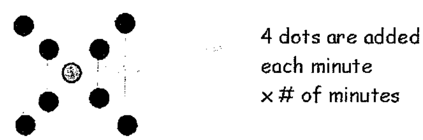
Session One
Conceptualizing and Representing Linear Relationships
Solution Methods: Growing Dots 1

t = number of minutes n = number of arms

Closed/Explicit Methods

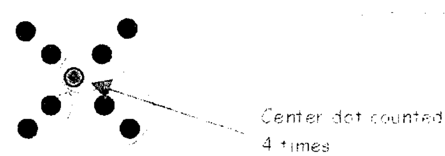


1. Arms + center: $t4 + 1 \Longrightarrow$ Generalized form: $tn + 1$
   Total includes the number of dots in each arm
   x the number of arms and add the beginning dot.

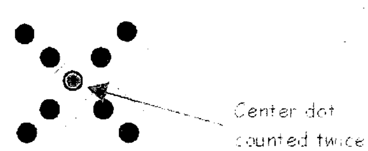


2. Groups of 4 + center: $4t + 1 \Longrightarrow$ Generalized form: $nt + 1$
   Total includes the number of dots added for each minute
   x the number of minutes + the beginning dot.

3. Extended arms: $(t + 1)4 - 3 \Longrightarrow$ Generalized form: $(t + 1)n - 3$
   Total includes the number of dots in each arm x
   the number of arms.
   The center (beginning) dot is counted 4 times,
   so 3 must be subtracted (excluded) from the total.



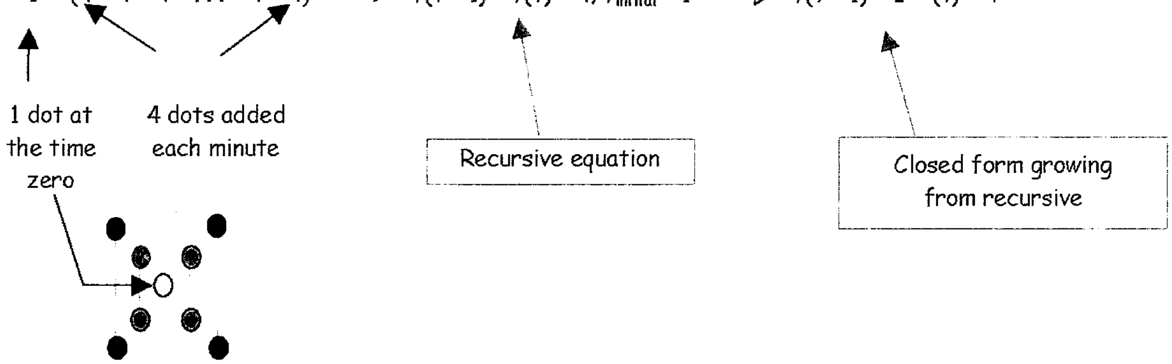
4. Crossed arms: $2(2t + 1) - 1$
   An arm is defined as all the dots in a line.
   The center dot is counted twice, so 1 must be
   subtracted (excluded) from the total.
   (Note: this method doesn't generalize for all configurations.)



Recursive Method

1. **Beginning with one dot, four new dots are added each minute. Number of dots at time 0 plus the number of dots added each minute times the number of minutes (t).**

$1 + (4 + 4 + 4 + \ldots + 4 + 4) \Longrightarrow f(t + 1) = f(t) + 4;\ f_{initial} = 1 \Longrightarrow f(t + 1) = 1 + (t) \bullet 4$



Session One
Conceptualizing and Representing Linear Relationships

Transcript
Growing Dots 1 Lesson, Period 4: Danielle's and James' Methods

[$5\frac{1}{2}$ minutes]

15:32	**Kirk:**	All right, can I get your attention for just a moment please? Um . . .
15:36	**Kirk:**	The first part of the question says describe the pattern. And we talked about, someone said it looks like an X pattern.
15:42	**Kirk:**	I've also heard people tossing around, you add four to each one, different things like that.
15:49	**Kirk:**	The second part of that question says, assuming the sequence continues how many dots are there at 100 minutes?
15:54	**Kirk:**	Can I get somebody to share with the class what they got for 100 minutes and how you arrived at your answer?
16:01	**Kirk:**	Danielle?
16:01	**Danielle:**	Four hundred and one.
16:02	**Kirk:**	Okay, she says 401. Anybody else agree with that?
16:05	**James:**	Agree?
16:07	**Kirk:**	Can you explain how you arrived at 401?
16:09	**Danielle:**	Because I got the equation x times four plus one. The plus one being the center, um, x being the dots around it or 100 and four being all the dots except the center.

Developing Mathematical Ideas

Deborah Schifter
Virginia Bastable
Susan Jo Russell

Seven DMI modules

- Building a system of tens
- Making meaning for operations
- Examining features of shape
- Measuring space in one, two, and three dimensions
- Working with data
- Reasoning algebraically about operations
- Patterns, functions, and change

36 + 17

a. (30 + 6) + (10 + 7) = (30 + 10) + (6 + 7) = (40 + 13).

b. (36 + 17) = (33 + 20) Move 3 from the 36 to the 17.
or
(36 + 17) = (40 + 13) Move 4 from the 17 to the 36.

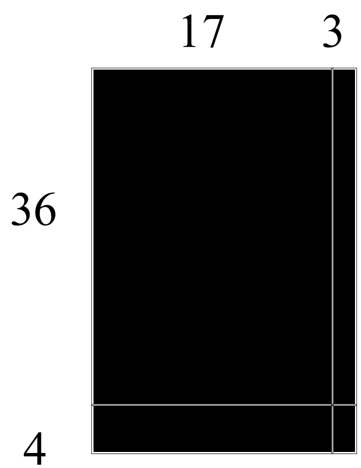
c. Round up to (40 + 20). Then subtract the extra 4 and the extra 3

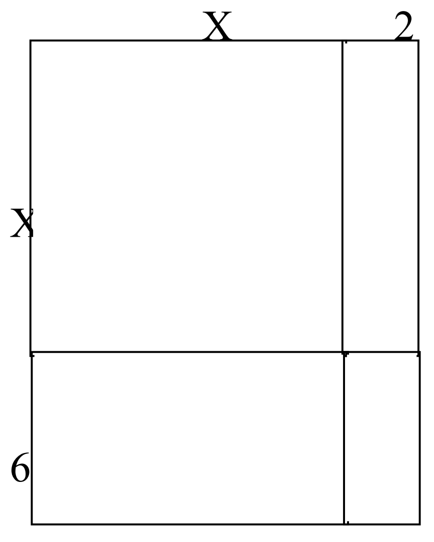
36 x 17

a. Why isn't (30 + 6) x (10 + 7) = (30 x 10) + (6 x 7)?

b. Why isn't (36 x 17) = (33 x 20)?

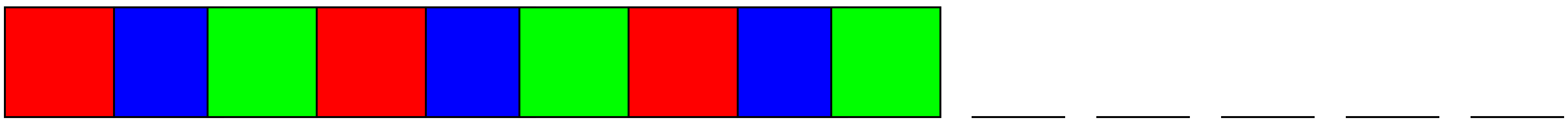
c. Why isn't (36 x 17) = (40 x 20) - 4 - 3 ?


17
3
36
4

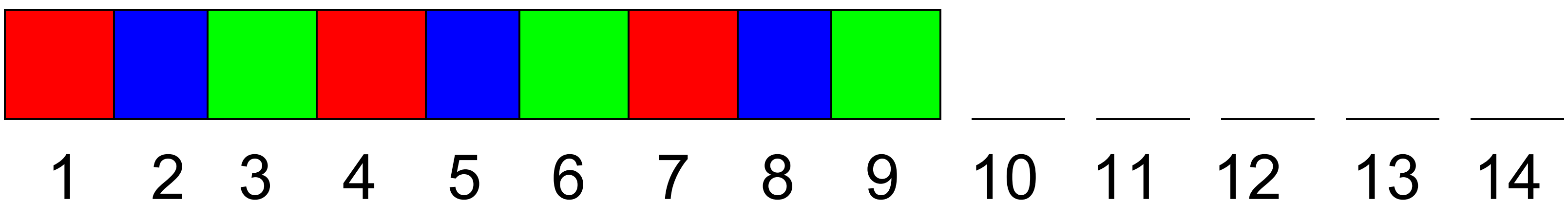


$$(x + 2)\,(x + 6) = x^2 + 2x + 6x + 12 = x^2 + 8x + 12$$

[redacted]



The repeating pattern of red, blue, green, red, blue, green continues in the same way.



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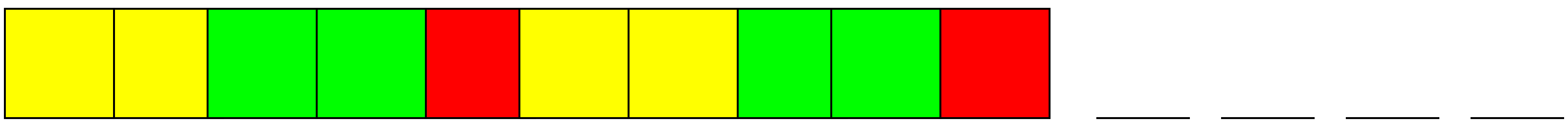
Red, blue, green repeating pattern

Number of green square	Position on the number strip
1	3
2	6
3	9
4	12
5	15
10	?
20	?

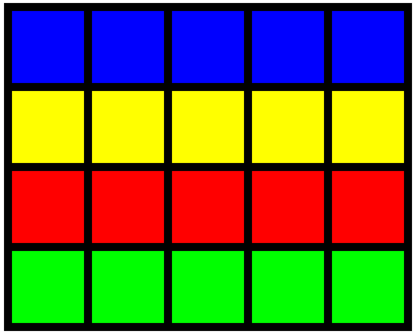
Red, blue, green repeating pattern

Number of red square	Position on the number strip
1	1
2	4
3	7
4	10
5	13
10	?
20	?



The repeating pattern of yellow, yellow, green, green, red

continues in the same way.



This building has five rooms on each floor.

Session One
Conceptualizing and Representing Linear Relationships

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16:09	**Danielle:**	Because I got the equation x times four plus one. The plus one being the center, um, x being the dots around it or 100 and four being all the dots except the center.

18:12	**Kirk:**	Hold on.
18:13	**James:**	Think that, I didn't think . . .
18:13	**Kirk:**	Then I won't be able to use these neat things.
18:17	**James:**	Well, I don't have to go up there, there's no reason.
18:19	**Kirk:**	There's no reason to come up here?
18:21	**James:**	No, I can just tell you.
18:23	**James:**	'Cause I could be wrong. That's bad on my part.
18:26	**Kirk:**	So what if you're wrong?
18:27	**James:**	No, it ain't like that.
18:28	**Kirk:**	Big deal.
18:30	**Kirk:**	I'm missing a dot. Do you mind, Danielle? No offense.
18:37	**Kirk:**	Um . . . Are you willing to come up?
18:39	**S:**	No.
18:40	**Kirk:**	You don't want to come up either?
18:42	**James:**	I'll tell you.
18:42	**Kirk:**	Just draw. Come on, James.
18:48	**Kirk:**	Yours might even be different than his.
18:49	**James:**	I was thinking this.
18:53	**James:**	This right here, I wasn't worried about this in the center. I didn't really think of that as one. I was thinking like . . .
19:02	**James:**	This says like the sequence or whatever. Well, this started at one and over here is five and nine over here. I just took they added four every time.
19:10	**James:**	See like four, four, four. And that's why I got x plus four for the equation.
19:19	**Kirk:**	So your equation is x plus four. It's not what Danielle had.
19:22	**Brandie:**	No.

20:20	**Kirk:**	I am curious about this because at one minute, what would x be in your equation?
20:27	**James:**	To me? x? Like the number, the number that I'm adding it to, like what was in the previous picture. What I'm adding to.
20:34	**Kirk:**	Oh. OK. So you're saying that in the previous one, the previous picture . . .
20:37	**Brandie:**	Would be x.
20:38	**James:**	The previous one would be x. And then that plus four is the next picture.
20:47	**Kirk:**	I see. I thought you meant x was minutes. 'Cause in Danielle's problem, correct me if I'm wrong, Danielle, for your problem, could have I replaced x with a t?
20:59	**Kirk:**	Were you saying that this was the number of minutes?
21:04	**Danielle:**	Um . . .
21:04	**Kirk:**	In your equation. Or am I wrong?
21:05	**Danielle:**	Yes.
21:08	**Janelle:**	You're wrong.
21:08	**Kirk:**	OK, Danielle was saying it was the number of minutes? Now James is saying his is the previous plus four.

19:23	**Kirk:**	Hm.
19:24	**Kirk:**	And it worked? The x plus four worked for you?
19:27	**Kirk:**	Did you get 401 dots? Or did you get a different number?
19:29	**James:**	Four hundred dots.
19:30	**Kirk:**	You got 400. Okay.
19:33	**James:**	'Cause she counted the center, we didn't count the center like she did.
19:36	**Kirk:**	Okay, and why didn't you count the center?
19:38	**Kirk:**	Just . . .
19:39	**James:**	Well . . .
19:41	**Brandie:**	'Cause the center is not growing, its just what's growing around it.
19:44	**James:**	It wasn't, I think it wasn't part of it. If you count the center, then that's saying that this is adding five. I can't say is adding four to make this.
19:52	**James:**	'Cause if you start one and you get five over here, one plus four is five.
19:57	**Kirk:**	Uh-huh.
19:58	**James:**	Right?
19:58	**Kirk:**	Uh-huh.
19:59	**James:**	So, I didn't count this middle one.
20:20	**Kirk:**	OK. Is the middle one part of the picture, though?
20:02	**James:**	Yes.
20:03	**Kirk:**	So, technically if you did it your way, it might be worth going back and adding the middle one since it's part of the picture.
20:10	**Brandie:**	But the middle one is never changing.
20:11	**Kirk:**	One other thing I want to . . . OK, thank you, James. Uh, one other thing I want to comment on, this x plus four.

16:21	**Kirk:**	Okay. Can you illustrate that with that little diagram there?
16:29	**Kirk:**	Actually it's a big diagram, it's not that little.
16:40	**Danielle:**	This is the center or one in the equation.
16:44	**Danielle:**	And, four would be all the dots except the center.
16:50	**Danielle:**	It would be like these two and these two.
16:52	**Kirk:**	Can you draw that on there?
16:54	**Kirk:**	What you mean?
17:14	**Danielle:**	And x is how many dots out from the center? Four ends.
17:21	**Danielle:**	Like a circle. It's like a circle going around.
17:28	**Danielle:**	Like that. And x would be this one and going out.
17:35	**Kirk:**	Excellent.
17:37	**Kirk:**	Give her a hand. Good explanation.
17:43	**Kirk:**	Anybody else have four x plus one as their rule?
17:46	**Kirk:**	Can I get somebody who maybe sees this a little bit differently? I like the way she explained that a lot.
17:51	**Kirk:**	Anybody see it differently than, um . . . She's got these . . . you are calling them circles, right?
17:57	**Danielle:**	Yeah.
17:57	**Kirk:**	Could they kind of be squares?
17:59	**James:**	I see it different.
18:10	**Danielle:**	They could.
18:28	**Kirk:**	Circles or squares. She sees it as growing out. Anybody see it a different way? OK.
18:05	**James:**	I ain't gonna to draw it, I can just tell you.
18:07	**Kirk:**	You are not going to draw it?
18:08	**James:**	I'll just tell you I see it different cause I didn't think . . .