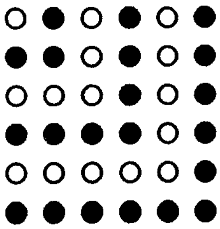
PICTURE PROOFS

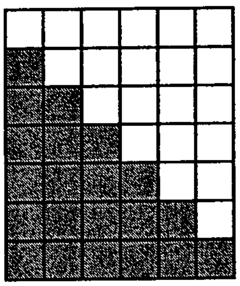
STEP 1: Each of these pictures shows an explanation of one of the formulas. Match the formula to the picture.

1



A $1+2+3+4+\cdots+n=\frac{1}{2}n\cdot(n+1)$

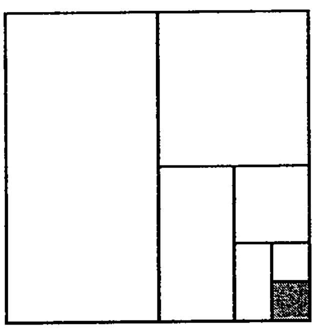
2



B $1+3+5+7+\cdots+(2n-1)=n^2$

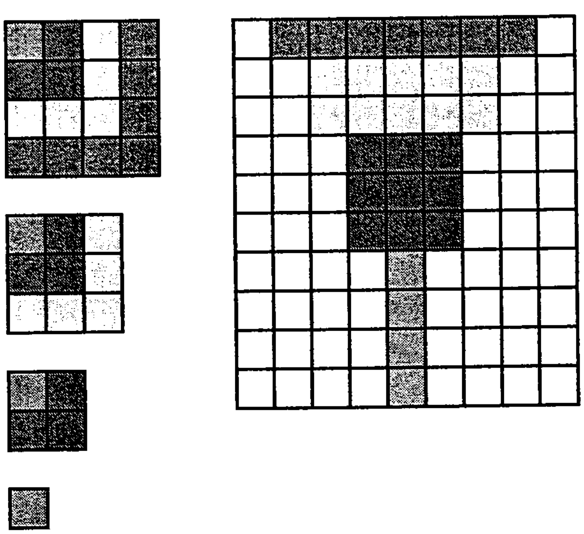
C $1+2^2+3^2+4^2+\cdots+n^2=\frac{1}{6}n\cdot(n+1)(2n+1)$

3



D $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{2^n}=2-\frac{1}{2^n}$

4



STEP 2: What is the value of *n* represented in the picture? Draw the picture for the next larger value of *n*.

STEP 3: Explain how the picture represents the left-hand side of the equation. Explain how it represents the right-hand side.

STEP 4: For each one, make a table of values, including both the left-hand and right-hand side of the equation, with *n=1, 2, 3, 4, 5, 6, 7,* or more

from
Saunders' presentation.

III. Area and Integrals

Week 7. Areas of regions bounded by curves

Review: Billstein:
Section 11-2: #15, #21d), #25, #32,#39, #42, #43

Problem of the Week: Billstein Circle Problem
Section 11-2: #21 (d)

Monday: Scaling area

The stretching area principle: If an geometric figure is stretched or shrunk by a factor of r in one direction, then the area increases (decreases) by a factor of r.

The scaling area principle: If an geometric figure is enlarged (or reduced) by a factor of r, then the area increases (decreases) by a factor of r^2.

138. Use the stretching area principle to conclude the area of a rectangle, assuming you know the area of a square.

139. Explain why the scaling principle is true for a square.

140. An ellipse is given by an equation of the form: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Use scaling arguments to find a formula for the area of an ellipse.

141. Use the scaling area principle to explain: The ratio of the areas of two circles is the ratio of the squares of their radii.

142. Using the scaling area principle rather than computing areas with formulas, do these problems from Billstein Section 11-2 #21e-g, #32, #39, #43

143. Get the menu for your favorite pizza place. Figure out a formula for the price of their plain cheese, thin crust pizza as a function of the diameter of the pizza. Is the price proportional to the diameter or is it proportional to the area or is there a combination?

Wednesday: Finding areas with the calculator

144. Use your calculator to compute the area of a rectangle with dimensions 3X7.

145. Use your calculator to find the area of a right triangle if the two legs have length 5cm and 3cm.

146. Use the absolute value function on your calculator to find the approximate area of an equilateral triangle that has sides length 10 inches.

147. Use your calculator to find the approximate area of a circle that has radius 3 centimeters.

148. Confirm the formula of the area of an ellipse by computing examples with your calculator.

Thursday: Area of a circle: another limits problem

149. Find an approximate the area of a circle by covering the circles with units squares and counting the squares.

150. Write a program to approximate the area of a circle that has radius 1 without using the formula for the area of a circle or using the value of π.

151. How would you modify the program to approximate the area of a circle that has radius r?

152. Modify your program to approximate the area of the ellipse given by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Check your answer against the formula found in problem 140.

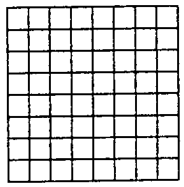
Week 8. Parabolas and Area

Review: From Week 1

Formula for the sum of squares

Problem of the Week: How Many Squares?

There are 64 1x1 squares in this checkerboard. How many squares of all different sizes can you find in this picture?



Monday: The area bounded, in part, by a parabola.

153. Approximate the area bounded by $y = x^2$, the x-axis and the line $x = 1$ using four subdivisions of the interval $[0,1]$ on the x-axis.

154. Approximate the area bounded by $y = x^2$, the x-axis and the line $x = 1$ using five subdivisions of the interval $[0,1]$ on the x-axis.

155. Approximate the area bounded by $y = x^2$, the x-axis and the line $x = 1$ using six subdivisions of the interval $[0,1]$ on the x-axis.

156. Find an expression for the area bounded by $y = x^2$, the x-axis and the line $x = 1$ using n subdivisions of the interval $[0,1]$ on the x-axis.

Answer these questions and then use your calculator to confirm your answers.

157. Find the area bounded by $y = x^2$, the x-axis and the line $x = 1$. In other words, compute $\int_0^1 x^2 dx$.

158. What is the area under $f(x) = x$ on the interval [0,1]? In other words, compute $\int_0^1 x dx$

159. What is the area under $f(x) = 1 - x^2$ on the interval [0,1]? In other words, compute $\int_0^1 (1 - x^2)dx$

160. What is the area under $f(x) = 1 - x^2$ on the interval [-1,1]? In other words, compute $\int_{-1}^1 (1 - x^2)dx$

161. What is the area under $f(x) = x^2 + 3$ on the interval [0,1]? In other words, compute $\int_0^1 (x^2 + 3)dx$

162. What is the area under $f(x) = 3x^2$ on the interval [0,1]? In other words, compute $\int_0^1 3x^2 dx$

Answer these questions and then use your calculator to confirm your answers. It will help to first draw a careful graph of the function

163. Compute $\int_1^3 4x + 1 dx$

164. Compute $\int_{-1}^1 |x| dx$

165. Compute $\int_{-2}^0 (x^2 + 2x + 1) dx$

Wednesday: Changing scale in the x direction

166. What is the area under $f(x) = x^2$ on the interval [0,4]? In other words, compute $\int_0^4 x^2 dx$

167. Compute $\int_0^2 x^2 dx$

168. What is the area under $f(x) = x^2$ on the interval [0.5,1]? In other words, compute $\int_{0.5}^1 x^2 dx$

169. Find a formula for $\int_0^x t^2 dt$.

Thursday: Cavaleri and other principles of integrals

170. Evaluate $\int_0^1 (x^2 + x) dx$.

171. Find a formula for $\int_0^1 (ax^2 + bx + c) dx$.

172. Find a formula for $\int_A^B (x^2 + 3x + 5) dx$.

173. Using the formulas you developed, compute $\int_1^2 1 - x^2 dx$. Explain why the answer is negative.

174. Carefully graph the following functions shade the area(s) indicated by the integral and compute the integral:

a) $\int_{-1}^0 (2x^2 + 3x + 2) dx$.

b) $\int_0^4 (x^2 - 4x + 3) dx$.

c) $\int_{-1}^1 (3x^2 - 2x + 1) dx$.