Cap-and-Trade Schemes for the Emissions Markets: Design, Calibration and Option Pricing

René Carmona

MSRI May 4, 2009

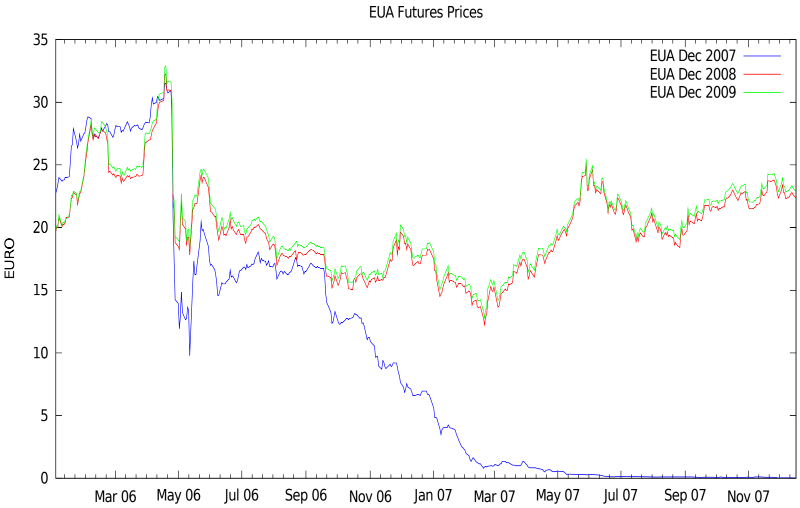
Cap-and-Trade Schemes for Emission Trading

- **Cap & Trade Schemes for CO_2 Emissions**
  - Kyoto Protocol
  - Mandatory Carbon Markets **(EU ETS, RGGI since 01/01/09)**
  - Lessons learned from the EU Experience
  - Cap-and-Trade vs Carbon Tax
  - Offsets and Clean Development Mechanism (CDM & JI)
- **Mathematical (Equilibrium) Models**
  - Price Formation for Goods and Emission Allowances
  - New Designs and Alternative Schemes
  - Calibration & Option Pricing
- **Computer Implementations**
  - Several case studies (Texas, Japan)
  - Practical Tools for Regulators and Policy Makers

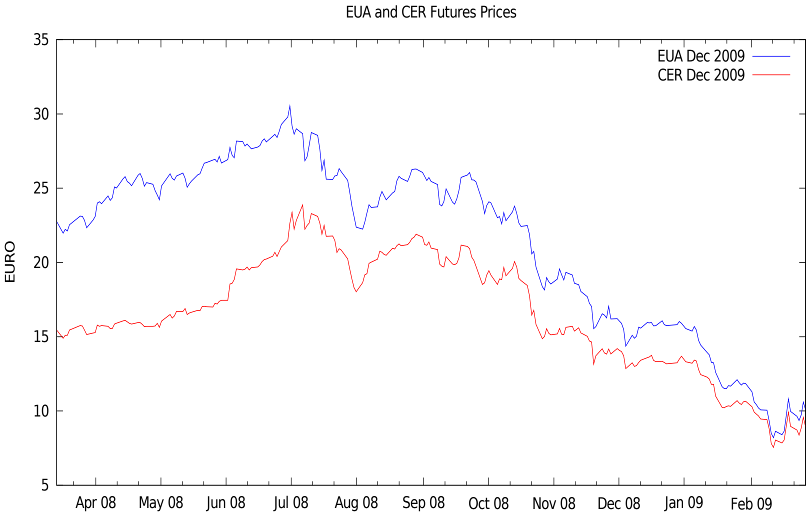
EU ETS First Phase: Main Criticism

- **No (Significant) Emissions Reduction**
  - DID Emissions go down?
  - Yes, but as part of an existing trend
- **Significant Increase in Prices**
  - Cost of Pollution passed along to the ”end-consumer”
  - Small proportion (40%) of polluters involved in EU ETS
- **Windfall Profits**
  - Cannot be avoided
  - Proposed Remedies
    - Stop Giving Allowance Certificates Away for Free !
    - Auctioning
    - Carbon Tax
- **Multi Compliance Periods**
  - Banking
  - Borrowing

Falling Carbon Prices: What Happened?



CDM: Can we Explain CER Prices?



Description of the Economy

- **Finite set** $\mathcal{I}$ of **risk neutral firms**
- **Producing a finite set** $\mathcal{K}$ of **goods**
- Firm $i \in \mathcal{I}$ can use **technology** $j \in \mathcal{J}^{i,k}$ to produce good $k \in \mathcal{K}$
- **Discrete time** $\{0, 1, \cdots, T\}$
- **No Discounting** Work with T-Forward Prices
- **Inelastic Demand**

$$\{D^k(t);\ t = 0, 1, \cdots, T-1,\ k \in \mathcal{K}\}.$$

- . . . .

Regulator Input (First Phase EU ETS)

Standard Cap-and-Trade Scheme

At inception of program (i.e. time $t = 0$)

- **INITIAL ALLOCATION** of **allowance certificates**

$$\theta_0^i \quad \text{to firm } i \in \mathcal{I}$$

- Set **PENALTY** π for emission unit **NOT** offset by allowance certificate at end of **compliance period**

Extensions (not discussed here)

- **Risk aversion** and agent preferences (existence theory easy)
- **Elastic** demand (e.g. smart meters for electricity)
- **Multi-period models** with lending, borrowing and withdrawal (more realistic)
- . . . .

Goal of Equilibrium Analysis

Find **two stochastic processes**

- **Price of one allowance**

$$A = \{A_t\}_{t \geq 0}$$

- **Prices of goods**

$$S = \{S_t^k\}_{k \in \mathcal{K},\, t \geq 0}$$

satisfying the usual conditions for the existence of a

competitive equilibrium

(to be spelled out below).

Individual Firm Problem

During each time period $[t, t+1)$

- Firm $i \in \mathcal{I}$ **produces** $\xi_t^{i,j,k}$ of good $k \in \mathcal{K}$ with technology $j \in \mathcal{J}^{i,k}$
- Firm $i \in \mathcal{I}$ **holds** a position θ_t^i in emission credits
- It **costs** firm $i \in \mathcal{I}$, $C_t^{i,j,k}$ to produce one unit of $k \in \mathcal{K}$ with technology $j \in \mathcal{J}^{i,k}$

$$
\begin{aligned}
L^{A,S,i}(\theta^i, \xi^i) := \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{J}^{i,k}} \sum_{t=0}^{T-1} (S_t^k - C_t^{i,j,k}) \xi_t^{i,j,k} \\
+ \theta_0^i A_0 + \sum_{t=0}^{T-1} \theta_{t+1}^i (A_{t+1} - A_t) - \theta_{T+1}^i A_T \\
- \pi(\Gamma^i + \Pi^i(\xi^i) - \theta_{T+1}^i)^+
\end{aligned}
$$

where

$$
\Gamma^i \text{ random}, \qquad \Pi^i(\xi^i) := \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{J}^{i,k}} \sum_{t=0}^{T-1} e^{i,j,k} \xi_t^{i,j,k}
$$

Problem for (risk neutral) firm $i \in I$

$$
\max_{(\theta^i, \xi^i)} \mathbb{E}\{L^{A,S,i}(\theta^i, \xi^i)\}
$$

In the Absence of Cap-and-Trade Scheme (i.e. $\pi = 0$)

If (A^*, S^*) is an equilibrium, the optimization problem of firm i is

$$\sup_{(\theta^i,\xi^i)} \mathbb{E}\left[\sum_{k\in\mathcal{K}}\sum_{j\in\mathcal{J}^{i,k}}\sum_{t=0}^{T-1}(S_t^k - C_t^{i,j,k})\xi_t^{i,j,k} + \theta_0^i A_0 + \sum_{t=0}^{T-1}\theta_{t+1}^i(A_{t+1} - A_t) - \theta_{T+1}^i A_T\right]$$

We have $A_t^* = \mathbb{E}_t[A_{t+1}^*]$ for all t and $A_T^* = 0$ (hence $A_t^* \equiv 0$!)

Classical competitive equilibrium problem where each agent maximizes

$$\sup_{\xi^i\in\mathcal{U}^i} \mathbb{E}\left[\sum_{k\in\mathcal{K}}\sum_{j\in\mathcal{J}^{i,k}}\sum_{t=0}^{T-1}(S_t^k - C_t^{i,j,k})\xi_t^{i,j,k}\right], \qquad (1)$$

and the equilibrium prices S^* are set so that supply meets demand. For each time t

$$((\xi_t^{*i,j,k})_{j,k})_i = \arg\max_{((\xi_t^{i,j,k})_{\mathcal{J}^{i,k}})_{i\in\mathcal{I}}} \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}^{i,k}} -C_t^{i,j,k}\xi_t^{i,j,k}$$

$$\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}^{i,k}} \xi_t^{i,j,k} = D_t^k$$

$$0 \le \xi_t^{i,j,k} \le \kappa^{i,j,k} \quad \text{for } i\in\mathcal{I}, j\in\mathcal{J}^{i,k}$$

Business As Usual (cont.)

The corresponding prices of the goods are

$$S_t^{*k} = \max_{i \in \mathcal{I},\, j \in \mathcal{J}^{i,k}} C_t^{i,j,k} \mathbf{1}_{\{\xi_t^{*i,j,k} > 0\}},$$

Classical **MERIT ORDER**

- At each time t and for each good k
- Production technologies ranked by increasing production costs $C_t^{i,j,k}$
- Demand D_t^k met by producing from the cheapest technology first
- Equilibrium spot price is the marginal cost of production of the most expensive production technoligy used to meet demand

Business As Usual
(typical scenario in Deregulated **electricity markets**)

Equilibrium Definition for Emissions Market

The processes $A^* = \{A^*_t\}_{t=0,1,\cdots,T}$ and $S^* = \{S^*_t\}_{t=0,1,\cdots,T}$ form an equilibrium if for each agent $i \in \mathcal{I}$ there exist strategies $\theta^{*i} = \{\theta^{*i}_t\}_{t=0,1,\cdots,T}$ (**trading**) and $\xi^{*i} = \{\xi^{*i}_t\}_{t=0,1,\cdots,T}$ (**production**)

- **(i) All financial positions are in constant net supply**

$$\sum_{i \in I} \theta^{*i}_t = \sum_{i \in I} \theta^i_0, \qquad \forall\, t = 0, \ldots, T+1$$

- **(ii) Supply meets Demand**

$$\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}^{i,k}} \xi^{*i,j,k}_t = D^k_t, \qquad \forall\, k \in \mathcal{K},\ \ t = 0, \ldots, T-1$$

- **(iii) Each agent $i \in I$ is satisfied by its own strategy**

$$\mathbb{E}[L^{A^*,S^*,i}(\theta^{*i}, \xi^{*i})] \geq \mathbb{E}[L^{A^*,S^*,i}(\theta^i, \xi^i)] \qquad \text{for all } (\theta^i, \xi^i)$$

Necessary Conditions

Assume

- (A^*, S^*) is an equilibrium
- (θ^{*i}, ξ^{*i}) optimal strategy of agent $i \in I$

then

- The allowance price A^* is a **bounded martingale** in $[0, \pi]$
- Its terminal value is given by

$$A^*_T = \pi \mathbf{1}_{\{\Gamma^i + \Pi(\xi^{*i}) - \theta^{*i}_{T+1} \geq 0\}} = \pi \mathbf{1}_{\{\sum_{i \in \mathcal{I}} (\Gamma^i + \Pi(\xi^{*i}) - \theta^{*i}_0) \geq 0\}}$$

- The **spot prices** S^{*k} of the goods and the **optimal production strategies** ξ^{*i} are given by the **merit order** for the equilibrium with **adjusted costs**

$$\tilde{C}^{i,j,k}_t = C^{i,j,k}_t + e^{i,j,k} A^*_t$$

Social Cost Minimization Problem

- **Overall production costs**

$$C(\xi) := \sum_{t=0}^{T-1} \sum_{(i,j,k)} \xi_t^{i,j,k} C_t^{i,j,k}.$$

- **Overall cumulative emissions**

$$\Gamma := \sum_{i \in I} \Gamma^i \qquad \Pi(\xi) := \sum_{t=0}^{T-1} \sum_{(i,j,k)} e^{i,j,k} \xi_t^{i,j,k},$$

- **Total allowances**

$$\theta_0 := \sum_{i \in I} \theta_0^i$$

The **total social costs from production and penalty payments**

$$G(\xi) := C(\xi) + \pi(\Gamma + \Pi(\xi) - \theta_0)^+$$

We introduce the global optimization problem

$$\xi^* = \arg \inf_{\xi \text{meets demands}} \mathbb{E}[G(\xi)],$$

Social Cost Minimization Problem (cont.)

- **First Theoretical Result**
  - There exists a set $\xi^* = (\xi^{*i})_{i\in I}$ realizing the minimum social cost
- **Second Theoretical Result**

(i) If $\overline{\xi}$ minimizes the social cost, then the processes $(\overline{A}, \overline{S})$ defined by

$$\overline{A}_t = \pi \mathbb{P}_t\{\Gamma + \Pi(\overline{\xi}) - \theta_0 \geq 0\}, \qquad t = 0, \dots, T$$

and

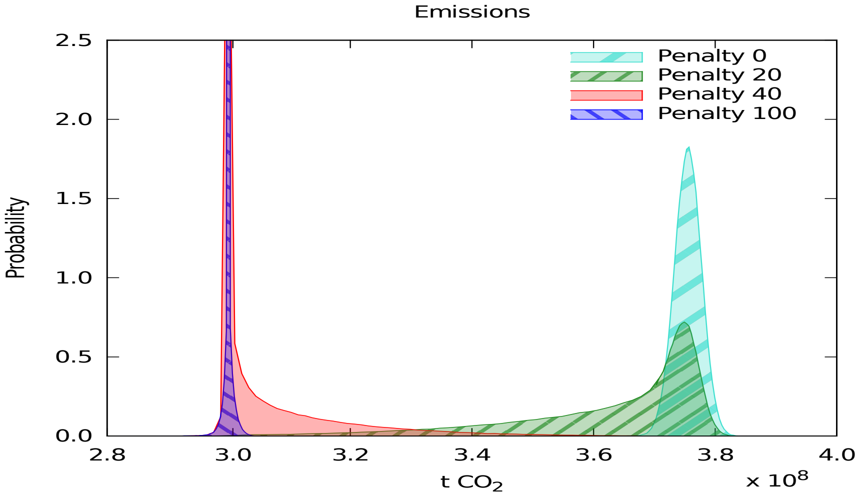
$$\overline{S}_t^k = \max_{i\in I,\, j\in J^{i,k}} (C_t^{i,j,k} + e_t^{i,j,k}\overline{A}_t)1_{\{\overline{\xi}_t^{i,j,k}>0\}}, \qquad t = 0, \dots, T-1 \quad k \in K,$$

form a **market equilibrium** with associated production strategy $\overline{\xi}$

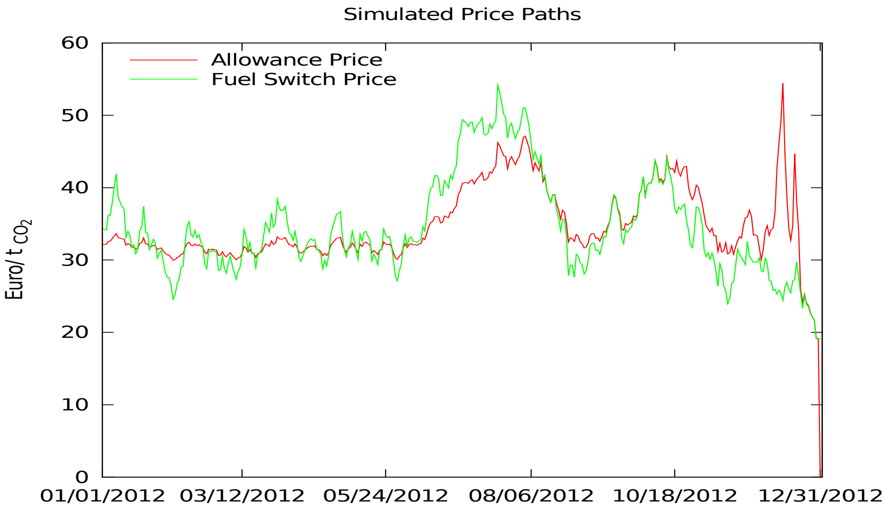
(ii) If (A^*, S^*) is an equilibrium with corresponding strategies (θ^*, ξ^*), then ξ^* solves the **social cost minimization problem**

(iii) The equilibrium allowance price is **unique**.

Effect of the Penalty on Emissions



Price Equilibrium Sample Path



Costs in a Cap-and-Trade

- **Consumer Burden**

$$\text{SC} = \sum_t \sum_k (S_t^{k,*} - S_t^{k,BAU*}) D_t^k.$$

- **Reduction Costs** (producers' burden)

$$\sum_t \sum_{i,j,k} (\xi_t^{i,j,k*} - \xi_t^{BAU,i,j,k*}) C_t^{i,j,k}$$

- **Excess Profit**

$$\sum_t \sum_k (S_t^{k,*} - S_t^{k,BAU*}) D_t^k - \sum_t \sum_{i,j,k} (\xi_t^{i,j,k*} - \xi_t^{BAU,i,j,k*}) C_t^{i,j,k} - \pi (\sum_t \sum_{ijk} \xi_t^{ijk} e_t^{ijk} - \theta_0)^+$$

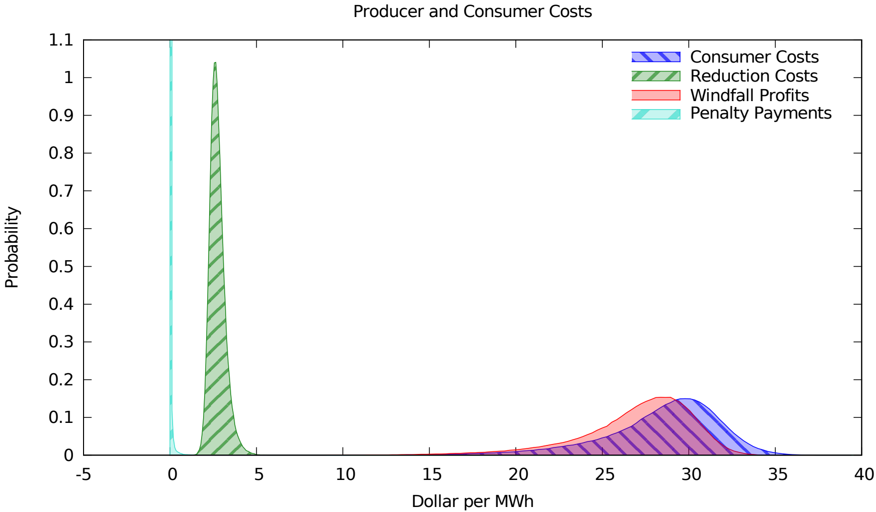
- **Windfall Profits**

$$\text{WP} = \sum_{t=0}^{T-1} \sum_{k \in K} (S_t^{*k} - \hat{S}_t^k) D_t^k - MA_0$$

where M is the number of allowances auction out, and

$$\hat{S}_t^k := \max_{i \in I, j \in J^{i,k}} C_t^{i,j,k} \mathbf{1}_{\{\xi_t^{*i,j,k} > 0\}}.$$

Costs in a Cap-and-Trade Scheme



Histograms of consumer costs, social costs, windfall profits and penalty payments of a standard cap-and-trade scheme calibrated to reach the emissions target with 95% probability and BAU.

One of many Possible Generalizations

Introduction of **Taxes** / **Subsidies**

$$\ddot{L}^{A,S,i}(\theta^i,\xi^i) = -\sum_{t=0}^{T-1} G_t^i + \sum_{k\in K}\sum_{j\in J^{i,k}}\sum_{t=0}^{T-1}(S_t^k - C_t^{i,j,k} - H_t^{j,k})\xi_t^{i,j,k}$$
$$+\sum_{t=0}^{T-1}\theta_t^i(A_{t+1}-A_t) - \theta_T^i A_T$$
$$-\pi(\Gamma^i + \Pi^i(\xi^i) - \theta_T^i)^+.$$

In this case

- In equilibrium, **production** and **trading** strategies remain the same $(\theta^\dagger,\xi^\dagger) = (\theta^*,\xi^*)$
- **Abatement costs** and **Emissions reductions** are also the same
- New equilibrium prices $(A^\dagger, S^\dagger)$ given by

$$A_t^\dagger = A_t^* \quad \text{for all } t = 0,\dots,T \quad \textbf{ALWAYS}$$
$$S_t^{\dagger k} = S_t^{*k} + H_t^k \quad \text{for all } k\in K, t=0,\dots,T-1 \quad \textbf{if } H_t^{j,k} = H_t^k$$

- Cost of the tax passed along to the end consumer

Alternative Market Design

- **Currently Regulator Specifies**
  - Penalty π
  - Overall Certificate Allocation θ_0 $(= \sum_{i \in I} \theta_0^i)$
- **Alternative Scheme (Still) Controlled by Regulator**
  (i) **Sets penalty level** π
  (ii) **Allocates allowances**
    - θ_0' at inception of program $t = 0$
    - then **proportionally to production**

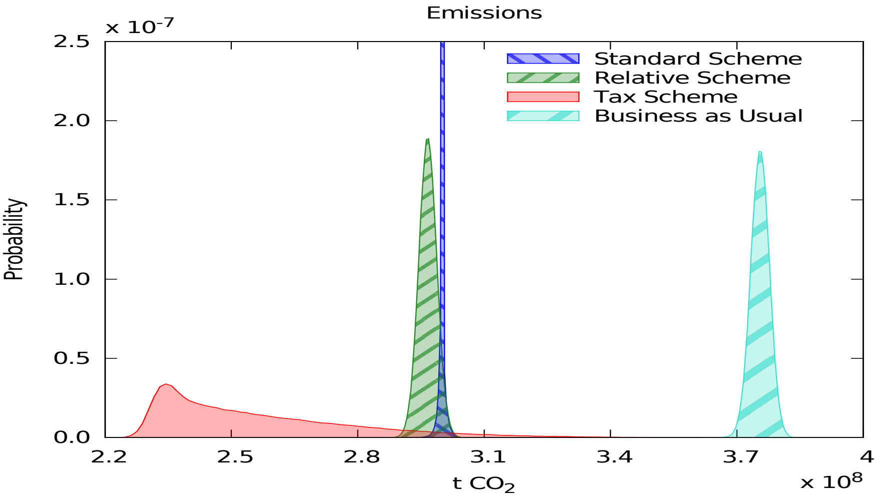
    $y\xi_t^{i,j,k}$ to agent i for producing $\xi_t^{i,j,k}$ of good k with technology j

  (iii) **Calibrates** y, e.g. in **expectation**.

$$y = \frac{\theta_0 - \theta_0'}{\sum_{t=0}^{T-1} \sum_{k \in K} \mathbb{E}\{D_t^k\}}$$

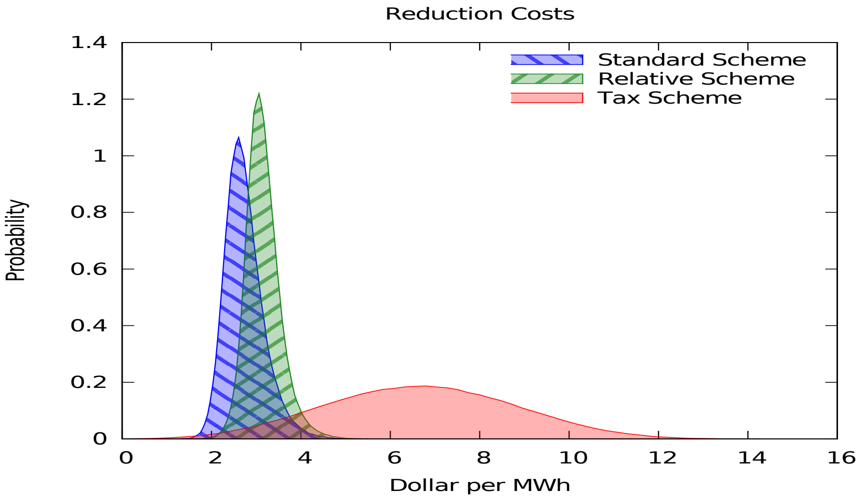
So total number of credit allowance is the same in expectation, i.e. $\theta_0 = \mathbb{E}\{\theta_0' + y \sum_{t=0}^{T-1} \sum_{k \in K} D_t^k\}$

Yearly Emissions Equilibrium Distributions



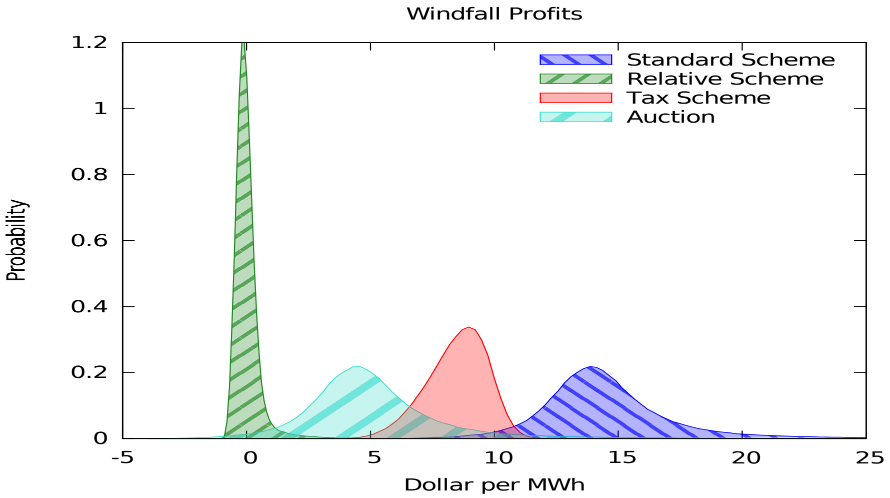
Yearly emissions from electricity production for the Standard Scheme, the Relative Scheme, a Tax Scheme and BAU.

Abatement Costs



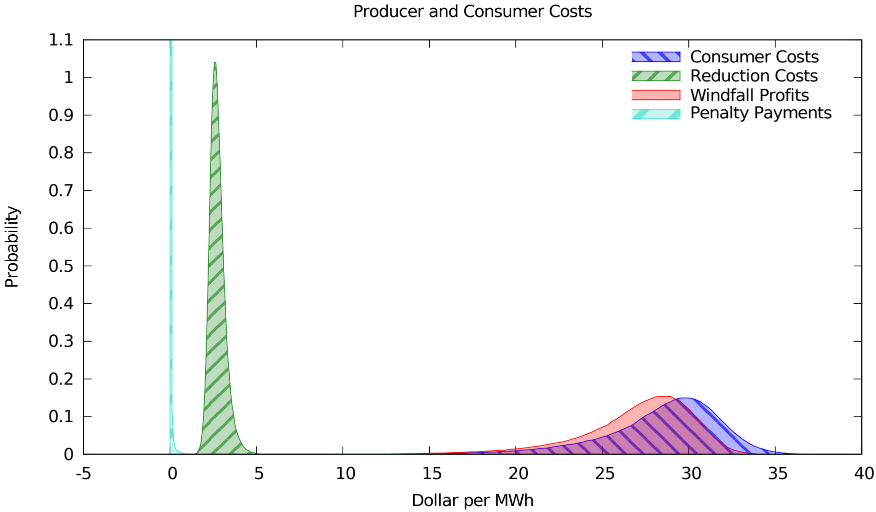
Yearly abatement costs for the Standard Scheme, the Relative Scheme and a Tax Scheme.

Windfall Profits



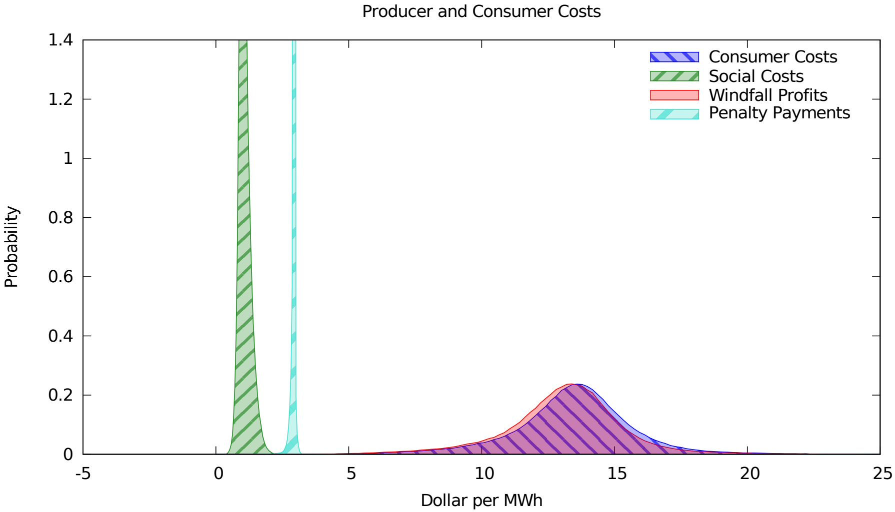
Histograms of the yearly distribution of windfall profits for the Standard Scheme, a Relative Scheme, a Standard Scheme with 100% Auction and a Tax Scheme

Japan Case Study: Windfall Profits



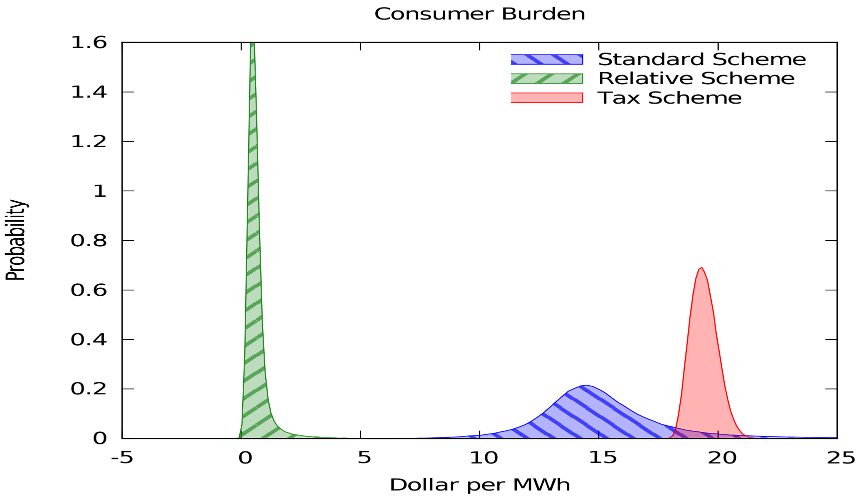
Histograms of the difference of consumer cost, social cost, windfall profits and penalty payments between BAU and a standard trading scheme scenario with a cap of 300Mt CO_2. Notice that taking into account fuel switching even

Japan Case Study: More Windfall Profits



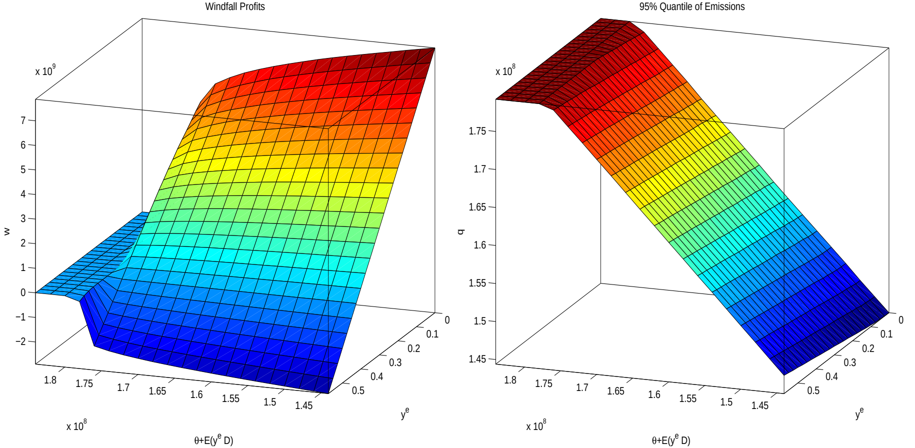
Histograms of the consumer cost, social cost, windfall profits and penalty payments under a standard trading scheme scenario with a cap of $330 MtCO_2$.

Japan Case Study: Consumer Costs



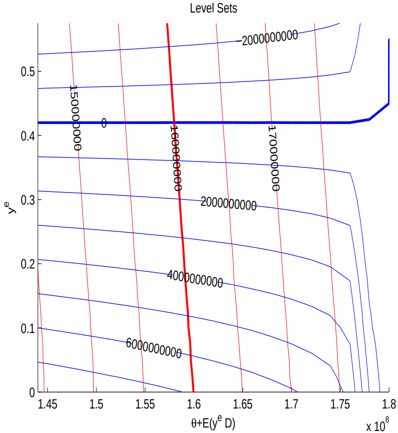
Histogram of the yearly distribution of consumer costs for the Standard Scheme, a Relative Scheme and a Tax Scheme. Notice that the Standard Scheme with Auction possesses the same consumer costs as the Standard

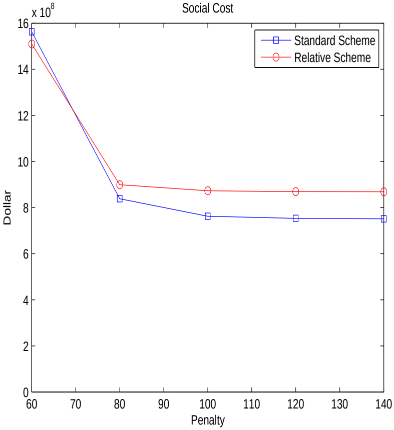
Numerical Results: Windfall Profits



Windfall profits (left) and 95% percentile of total emissions (right) as functions of the relative allocation parameter and the expected allocation

More Numerical Results: Windfall Profits





(left) Level sets of previous plots. (right) Production costs for electricity for one year as function of the penalty level for both the absolute and relative schemes.

Equilibrium Models: (Temporary) Conclusions

- Market Mechanisms **CANNOT** solve all the pollution problems
- **Cap-and-Trade Schemes CAN Work!**
  - Given the right emission target
  - Using the appropriate tool to allocate emissions credits
  - Significant Windfall Profits for Standard Schemes
- **Taxes**
  - Politically unpopular
  - Cannot reach emissions targets
- **Auctioning**
  - Fairness is Smoke Screen: Re-distribution of the cost
- **Relative Schemes**
  - **Pros**
    - Can Reach Emissions Target (statistics)
    - Possible Control of Windfall Profits
    - Minimize Social Costs
  - **Cons**
    - Number of Allowances **NOT** exactly known in advance

Hybrid Allocation Schemes

- **Partial Auctioning (Relative Scheme + Auction**
  - Same Pros as Relative Scheme
  - Number of Allowances **FIXED** in advance

Reduced Form Models & Option Pricing

- Emissions Cap-and-Trade Markets **SOON** to exist in the US
- **Option** Market **SOON** to develop
  - Underlying $\{A_t\}_t$ non-negative martingale with **binary terminal value**
  - Can think of A_t as of a binary option
  - Underlying of binary option should be *Emissions*
- Need for **Formulae** (closed or computable)
  - for Prices
  - for Hedges
- **Reduced Form Models**

Reduced Form Model for Emissions Abatement

- $\{X_t\}_t$ actual emissions at time t
  $dX_t = \sigma(t, X_t)dW_t - \xi_t dt$
  - ξ_t abatement (in ton of CO_2) at time t
  - $X_t = E_t - \int_0^t \xi_s ds$
    cumulative emissions in BAU minus abatement up to time t
- $\pi(X_T - K)^+$ penalty
  - T maturity (end of compliance period)
  - K regulator emissions' target
  - π penalty (40 EURO) per ton of CO_2 not offset by an allowance certificate
- **Social Cost** $\mathbb{E}\{\int_0^T C(\xi_s)ds + \pi(X_T - K)^+\}$
  - $C(\xi)$ cost of abatement of ξ ton of CO_2

Representative Agent Stochastic Control Problem

Informed Planner Problem

$$\inf_{\xi=\{\xi_t\}_{0\le t\le T}} \mathbb{E}\{\int_0^T C(\xi_s)ds + \pi(X_T - K)^+\}$$

Value Function

$$V(t,x) = \inf_{\{\xi_s\}_{t\le s\le T}} \mathbb{E}\{\int_t^T C(\xi_s)ds + \pi(X_T - K)^+ | X_t = x\}$$

HJB equation (e.g. $C(\xi) = \xi^2$)

$$V_t + \frac{1}{2}\sigma(t,x)^2 V_{xx} - \frac{1}{2}V_x^2$$

Calibration

Emission Allowance Price

$$A_t = V_x(t, X_t)$$

Emission Allowance Volatility

$$\sigma_A(t) = \sigma(t, X_t) V_{xx}(t, X_t)$$

Calibration ($\sigma(t)$ deterministic)

- Multiperiod (**Cetin. et al**)
- Close Form Formulae for Prices
- Close Form Formulae for Hedges

Reduced Form Models and Calibration

Allowance price should be of the form

$$A_t = \pi \mathbb{E}\{\mathbf{1}_N \,|\mathcal{F}_t\}$$

for a non-compliance set $N \in \mathcal{F}_t$. Choose

$$N = \{\Gamma_T \geq 1\}$$

for a random variable Γ_T representing the normalized emissions at compliance time. So

$$A_t = \pi \mathbb{E}\{\mathbf{1}_{\{\Gamma_T \geq 1\}} \,|\mathcal{F}_t\}, \qquad t \in [0, T]$$

We choose Γ_T in a parametric family

$$\Gamma_T = \Gamma_0 \exp\Big[\int_0^T \sigma_s dW_s - \frac{1}{2}\int_0^T \sigma_s^2\, ds\Big]$$

for some square integrable deterministic function

$$(0, T) \hookrightarrow \sigma_t$$

Dynamic Price Model for $a_t = \frac{1}{\pi} A_t$

- a_t is given by

$$a_t = \Phi\left(\frac{\Phi^{-1}(a_0)\sqrt{\int_0^T \sigma_s^2 ds} + \int_0^t \sigma_s dW_s}{\sqrt{\int_t^T \sigma_s^2 ds}}\right) \qquad t \in [0, T)$$

  where Φ is standard normal c.d.f.

- a_t solves the SDE

$$da_t = \Phi'(\Phi^{-1}(a_t))\sqrt{z_t}dW_t$$

  where the positive-valued function $(0, T) \ni t \hookrightarrow z_t$ is given by

$$z_t = \frac{\sigma_t^2}{\int_t^T \sigma_u^2 du}, \qquad t \in (0, T)$$

Aside: Binary Martingales as Underliers

Allowance prices are given by $A_t = \pi a_t$ where $\{a_t\}_{0 \le t \le T}$ satisfies

- $\{a_t\}_t$ is a martingale
- $0 \le a_t \le 1$
- $\mathbb{P}\{\lim_{t \to T} a_t = 1\} = 1 - \mathbb{P}\{\lim_{t \to T} a_t = 0\} = p$ for some $p \in (0, 1)$

The model

$$da_t = \Phi'(\Phi^{-1}(a_t))\sqrt{z_t}dW_t$$

suggests looking for martingales $\{Y_t\}_{0 \le t < \infty}$ satisfying

- $0 \le Y_t \le 1$
- $\mathbb{P}\{\lim_{t \to \infty} Y_t = 1\} = 1 - \mathbb{P}\{\lim_{t \to \infty} T_t = 0\} = p$ for some $p \in (0, 1)$

and do a **time change** to get back to the (compliance) interval $[0, T)$

Gives conditions for the SDE

$$da_t = \Theta(a_t)dW_t$$

for $x \hookrightarrow \Theta(x)$ satisfying

- $\Theta(x) > 0$ for $0 < x < 1$
- $\Theta(0) = \Theta(1) = 0$

to

- Converge to the boundaries 0 and 1
- NOT explode (i.e. NOT reach the boundaries in finite time)

Interestingly enough the solution of

$$dY_t = \Phi'(\Phi^{-1}(Y_t))dW_t$$

IS ONE OF THEM !

Explicit Examples

The SDE

$$dX_t = \sqrt{2}dW_t + X_t dt$$

has the solution

$$X_t = e^t(x_0 + \int_0^t e^{-s}dW_s)$$

and

$$\lim_{t\to\infty} X_t = +\infty \qquad \text{on the set } \{\int_0^\infty e^{-s}dW_s > -x_0\}$$

$$\lim_{t\to\infty} X_t = -\infty \qquad \text{on the set } \{\int_0^\infty e^{-s}dW_s < -x_0\}$$

Moreover Φ is **harmonic** so if we choose

$$Y_t = \Phi(X_t)$$

we have a martingale with the desired properties.

Another (explicit) example can be constructed from Ph. Carmona, Petit and Yor on Dufresne formula.

Historical Calibration

$$\{z_t(\alpha,\beta) = \beta(T-t)^{-\alpha}\}_{t\in[0,T]}, \qquad \beta > 0, \alpha \geq 1. \tag{2}$$

β is a multiplicative parameter

$$z_t(\alpha,\beta) = \beta z_t(\alpha,1), \quad t \in (0,T), \ \beta > 0, \quad \alpha \geq 1. \tag{3}$$

The function $\{\sigma_t(\alpha,\beta)\}_{t\in(0,T)}$ is given by

$$\sigma_t(\alpha,\beta)^2 = z_t(\alpha,\beta)e^{-\int_0^t z_u(\alpha,\beta)du} \tag{4}$$

$$= \begin{cases} \beta(T-t)^{-\alpha} e^{\beta\frac{T^{-\alpha+1}-(T-t)^{-\alpha+1}}{-\alpha+1}} & \text{for } \beta > 0, \alpha > 1 \\ \beta(T-t)^{\beta-1}T^{-\beta} & \text{for } \beta > 0, \alpha = 1 \end{cases} \tag{5}$$

Maximum Likelihood

Sample Data

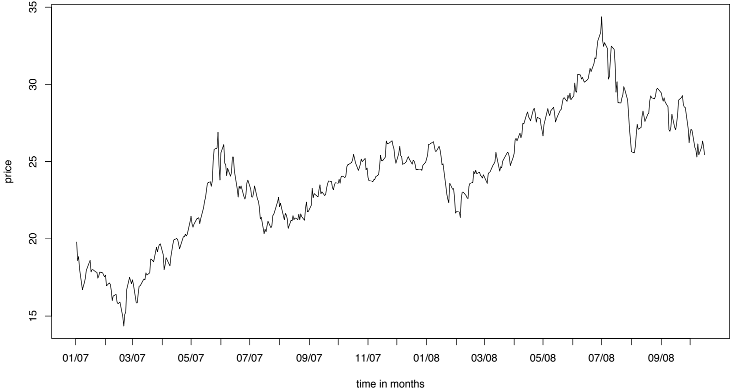


Figure: Future prices on EUA with maturity Dec. 2012

Call Option Price in One Period Model

for $\alpha = 1$, $\beta > 0$, the price of an European call with strike price $K \geq 0$ written on a one-period allowance futures price at time $\tau \in [0, T]$ is given at time $t \in [0, \tau]$ by

$$\begin{aligned} C_t &= e^{-\int_t^\tau r_s ds} \mathbb{E}\{(A_\tau - K)^+ \mid \mathcal{F}_t\} \\ &= \int (\pi\Phi(x) - K)^+ N(\mu_{t,\tau}, \nu_{t,\tau})(dx) \end{aligned}$$

where

$$\begin{aligned} \mu_{t,\tau} &= \Phi^{-1}(A_t/\pi)\sqrt{\left(\frac{T-t}{T-\tau}\right)^\beta} \\ \nu_{t,\tau} &= \left(\frac{T-t}{T-\tau}\right)^\beta - 1. \end{aligned}$$

Price Dependence on T and Sensitivity to β

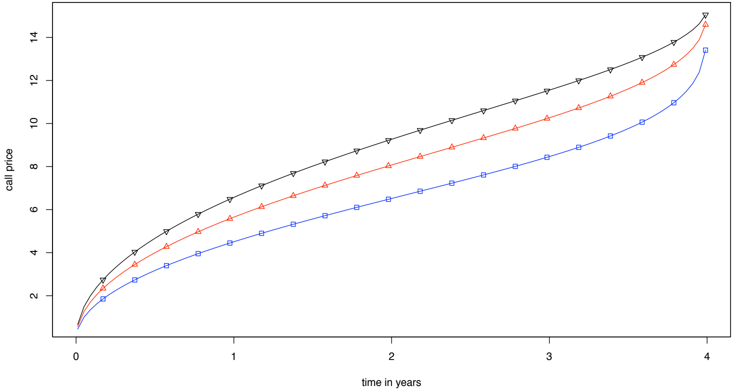


Figure: Dependence $\tau \mapsto C_0(\tau)$ of Call prices on maturity τ. Graphs $\square$, $\triangle$, and $\triangledown$ correspond to $\beta = 0.5$, $\beta = 0.8$, $\beta = 1.1$.

Presentations based on

- **Emissions Markets**
  - R.C., M. Fehr and J. Hinz: Mathematical Equilibrium and Market Design for Emissions Markets Trading Schemes. *SIAM J. Control and Optimization* (2009)
  - R.C., M. Fehr, J. Hinz and A. Porchet: Mathematical Equilibrium and Market Design for Emissions Markets Trading Schemes. *SIAM Review* (2009)
  - R.C., M. Fehr and J. Hinz: Properly Designed Emissions Trading Schemes do Work! (working paper)
  - R.C., M. Fehr and J. Hinz: Calibration and Risk Neutral Dynamics of Carbon Emission Allowances (working paper)
  - R.C. & M. Fehr: Relative Allocation and Auction Mechanisms for Cap-and-Trade Schemes (working paper)
  - R.C. & M. Fehr: The Clean Development Mechanism: a Mathematical Model. (in preparation)