Two questions for you

- If someone asked you to teach a mathematics course you have not taught before, what might be challenging for you?
- What kinds of knowledge might you wish you had before teaching the course for the first time?

Teacher knowledge and the complexity of orchestrating discussions in a Differential Equations classroom

Natasha M. Speer
University of Maine

Joseph F. Wagner
Xavier University

Two questions for you

- If someone asked you to teach a mathematics course you have not taught before, what might be challenging for you?
- What kinds of knowledge might you wish you had before teaching the course for the first time?

Overview for today

- Central questions:
  - What kinds of knowledge do teachers use?
  - What knowledge is needed to support the work teachers do while orchestrating discussions?
- Brief tour of the literature terrain
- Rationale for our approach to these questions
- Research design
- Data and findings
- Conclusions, implications, discussion

Things known from research on teaching

Teachers' course work in mathematics is not strongly correlated with higher achievement for their students.

(Begle, 1979; Monk, 1994)

"The conclusions of the few studies in this area are especially provocative because they undermine the certainty often expressed about the strong link between college study of a subject matter and teacher quality."

(Wilson, Floden, & Ferrini-Mundy, 2002, p. 191)

<u>Content knowledge is necessary but not sufficient</u>

Pedagogical Content Knowledge (PCK)

Especially useful examples, organization of content in a course, typical student difficulties, common strategies

(Shulman, 1986)

– influences practices

(Carpenter et al., 1988, 1989)

– influences student learning

(e.g., Fennema et al., 1996; Franke et al., 2001; 1997)

Other types of knowledge matter

Specialized Content Knowledge (SCK)

- *aka* "Mathematical knowledge for teaching"
  - used to do the "mathematical work" of teaching
  - to follow and understand students' mathematical thinking
  - to evaluate the validity of student-generated strategies

- Shown to play a role in teachers' practices and correlate with students' learning

(Ball & Bass, 2000; Hill et al 2004, 2005; Ma, 1999)

Appears to be important and distinct from "common" content knowledge

So…

- "What knowledge is needed for teaching?" is an extremely complicated question.
- Most related research involves elementary teachers.

What can you do when faced with a really complicated problem in mathematics?

Consult Pólya, use a heuristic:

> *Look for a related problem that is easier to solve and try to exploit its solution to solve the original problem*

Simplifying the "problem"

Mathematics teachers

Simplifying the “problem”

Mathematics teachers

With very strong content knowledge

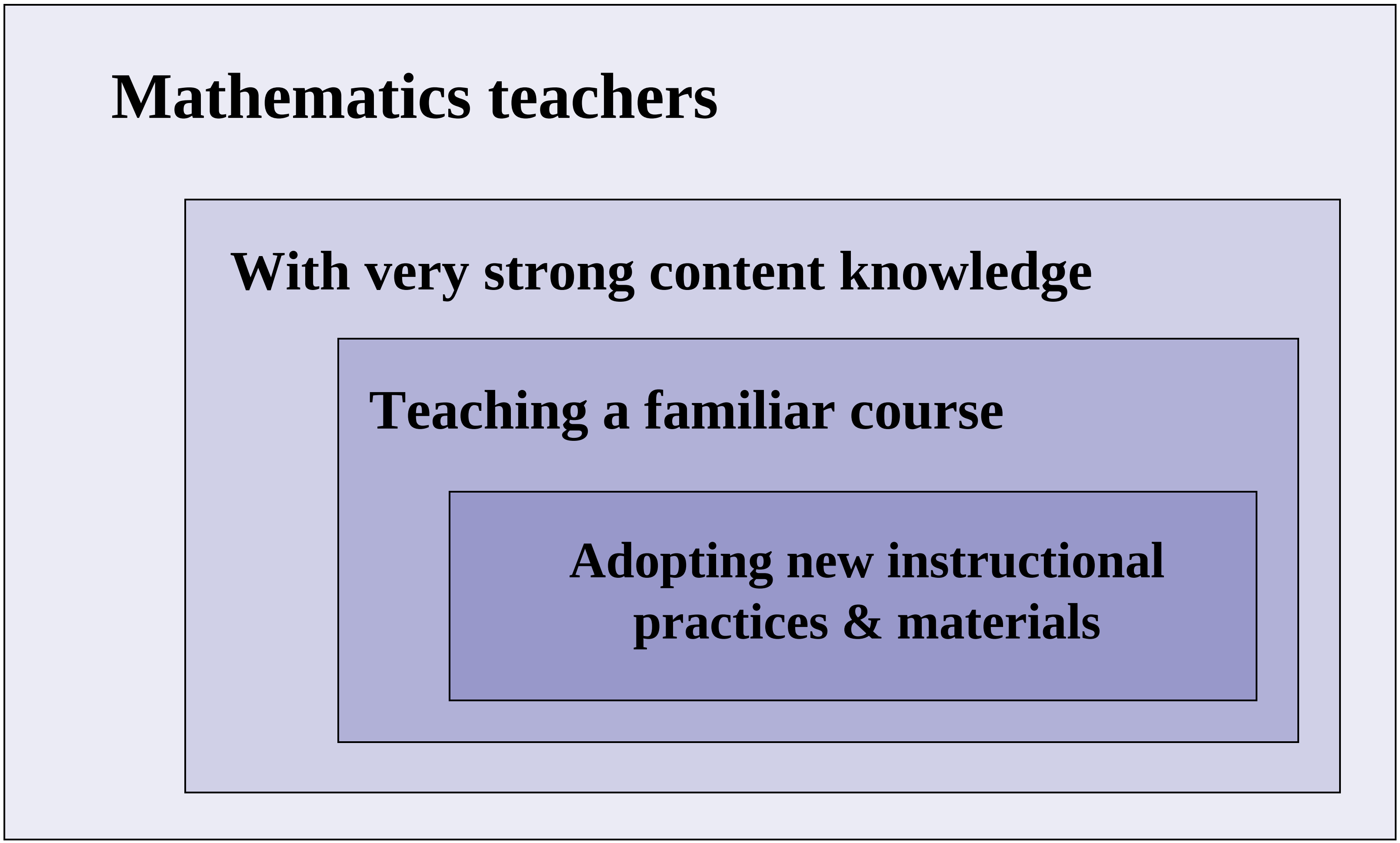
Simplifying the "problem"

Mathematics teachers

With very strong content knowledge

Teaching a familiar course

Simplifying the "problem"



Our "simplification"

- Very strong content knowledge
  - Prof. Gage, a research-active mathematician
- Teaching a familiar course
  - had taught differential equations many times
  - 17 years of university teaching experience
- Adopting new materials and practices
  - because he was dissatisfied with student learning
  - using "inquiry-oriented" DE materials for first time
  - relying on large-group discussions

Large-group discussions

- Important component of inquiry-based instruction
- Teachers need to
  - know/understand the mathematical goal(s)
  - keep the discussion moving
  - keep the discussion moving toward the mathematical goal(s)
- Doing this effectively is challenging

(Ball, 1993; Nathan & Knuth, 2003; Wagner, Speer, & Rossa, 2007; Williams & Baxter, 1996)

Scaffolding

Moves teachers make to keep the discussion moving in a productive direction

- Two types:
  - *Social scaffolding* = guiding the discussion
  - *Analytic scaffolding* = … in a (mathematically) productive direction

(Williams & Baxter, 1996)

- Ineffective instruction sometimes comes from an inappropriate balance of the two types
- Today: Focus on *knowledge* needed to provide analytic scaffolding

Analytic scaffolding

Component practices:

1. Recognize or figure out students' mathematical reasoning (both correct and incorrect)
2. Recognize or figure out how students' ideas have the potential to contribute to the mathematical goals of the discussion
3. Recognize or figure out how students' ideas are relevant to the development of students' understanding of the mathematics
4. Prudently select which contributions to pursue from among all those available

How do you decide a contribution may be productive?

- "Recognize it"
  - seen it before
  - read about it; been told about it
  - anticipated it (while you planned)
- "Figure it out" in the moment
  - pay attention to and hear students' contributions
  - follow and understand the mathematical ideas and/or ways of thinking that students offer
  - see how the mathematics in students' contributions connects to the mathematical goals of the discussion

How do you decide a contribution may be productive?

- "Recognize it" (PCK)
  - seen it before
  - read about it; been told about it
  - anticipated it (while you planned)
- "Figure it out" in the moment (SCK)
  - pay attention to and hear students' contributions
  - follow and understand the mathematical ideas and/or ways of thinking that students offer
  - see how the mathematics in students' contributions connects to the mathematical goals of the discussion

Specific research question

What does a teacher need to do and know to be able to recognize a student's contribution as productive?

Setting

- Introductory course in Differential Equations
  - medium-sized university in the Midwest
  - class of 21 students
- Inquiry-oriented curriculum developed by Rasmussen (2002), emphasizing conceptual understanding through group activities, problem solving, extensive discussion
- Mathematician as collaborator

Data *collection* methods

- Video recordings of (almost) all classes (1h 15m each)
- Audio recordings of debriefing sessions held after (almost) every class (45-60 minutes each)
- Prof. Gage's written reflections on his challenges, compiled at the end of the semester
- Observations of Prof. Gage's class during the prior semester
- Prof. Gage's written description of his prior teaching practices

Data *selection* methods

- Interview-inspired
  - selected days that Prof. Gage described as especially challenging
- Classroom-inspired
  - selected days that the researchers perceived as challenging for Prof. Gage
- Data used
  - transcripts from selected interviews and corresponding classroom videos
  - selected classroom videos and corresponding interview transcripts

Data *analysis* methods

- Transcribed selected classroom excerpts and interviews
- Used Grounded Theory-inspired approach to develop and test codes for evidence of challenges related to PCK and SCK
- Examined themes that emerged from the coding process
- Selected classroom episodes to share that were relatively simple to set up and describe

Four classroom episodes

Prof. Gage expressed his own frustration at not knowing how to respond to students.

- Scenes 1 & 3:

  When students pose good ideas

- Scenes 2 & 4:

  When students pose not-so-good ideas

Four classroom episodes

In each case, we examine

- what happened
- what might have happened differently had Prof. Gage "recognized" or "figured out" what was going on
- what components of analytic scaffolding were at play

Scene 1: (Not) Noticing the math

Students had been asked to propose a DE capable of modeling simple population growth under ideal circumstances.

This situation can be modeled with a rate of change equation:

$$\frac{dP}{dt} = something$$

What should the "*something*" be? Should the rate of change be stated in terms of just *P*, just *t*, of both *P* and *t*? Make a conjecture about the right hand side of the rate of change equation and provide reasons for your conjecture."

Scene 1: (Not) Noticing the math

- The activity was designed to lead students to recognize that
$$\frac{dP}{dt} = kP$$
was a reasonable model, with *P* representing the population size at time *t*, and *k* being a constant of proportionality.
- After discussing a variety of possible models, the students had narrowed their choices to two:
$$\frac{dP}{dt} = P \quad \text{and} \quad \frac{dP}{dt} = e^t$$

Scene 1 class transcript

G: If you take e^t, then if you differentiate it, you get it back. Some-. So are they the same or not? [*14 seconds silence*] How about, uh, you know, something so, like 2 times e^t? [*8 seconds silence*] $P(t)$ is 2 times e^t?

Prof Gage suggested that the students consider $P = 2e^t$ so they would:

- notice that $P = 2e^t$ solved only the first of the two DEs,
- see that $\frac{dP}{dt} = P$ had many solutions of the form $P = ke^t$, and as a result,
- begin to distinguish between the two models.

Scene 1 class transcript

G: If you take e^t, then if you differentiate it, you get it back. Some-. So are they the same or not? [*14 seconds silence*] How about, uh, you know, something so, like 2 times e^t? [*8 seconds silence*] $P(t)$ is 2 times e^t?

S: I really don't understand … what they mean by "the same."

G: OK. Can somebody may, uh, phrase what may be meant by 'these are the same'? Robert?

R: I think what was said is, OK, let's say you say $P(t)$ is where P is equal to e^t. Then if you take the derivative you'll get dP/dt is equal to e^t. But I think that's, I, while I can't deny the truth of that because you can just, by going back to the original equation, you can just substitute between P and e^t, you can derive the equation that says P is equal to e^t. I think that's a specific circumstance, that, you know, where that happens to work.

Scene 1 class transcript

G: Uh, sorry, where what happens to work?

R: That *P* is eq-, *P* is the same as e^t.

M: Can you come up with one where it doesn't work?

G: Yeah, can you come up with something where it wouldn't?

R: Well, let's say your, let's say your *P*(*t*) was *P+t*.

[*transcript omitted*]

G: OK, so Robert is saying that, you know, this kind of feels like some particular instance where something is happening, but we can't right now come up with something, ah, that kind of supports, supports that. Melanie?

Scene 1: Alternatives

- Someone recognizing the potential in Robert's suggestion might have
  - asked him to clarify what he meant by "that happens to work"
  - repeated his suggestion of $P = 2e^t$ and asked how it related to his point.
- Suggestion of $P = 2e^t$ was not picked up again for 20 minutes.
- What prevented Gage from picking up on the potential in Robert's contribution? What is needed to follow students' contributions in real time?

Scene 1: Interview transcript

During interviews, Prof. Gage repeatedly and often said that he felt unable to manage classroom discussion while still attending to students' thinking:

"I need to do too many things. I need to try to follow the train of thought carefully, and I need to try to figure out when is it a good place to do something. I have to look for certain clues that I am sensing are good ones to do something, and I – which is kind of a detached observation kind of thing. And the other is to be part of the thought process, to really follow it along, to try to direct it a little. There's a conflict there. I can't do both."

Scene 1: Summary

In order to "unpack" and make use of Robert's contribution, Gage would have had to

- infer meaning behind Robert's imprecise language, "that's a specific circumstance … where that happens to work"

  {*Component practice 1*}

- recognize that $P = 2e^t$ also "happens to work" for one but not the other of the equations

  {*Component practice 1*}

- judge how such a connection, once made, could be used to propel the class forward in a productive direction

  {*Component practice 2*}

Scene 2: (Not) Noticing the thinking

Building on an earlier discussion, students were asked to solve the following:

$$\frac{dP}{dt} = 2 - \frac{P}{10+t} \quad \rightarrow \quad (10+t)\frac{dP}{dt} + P = 2(10+t)$$

Scene 2: (Not) Noticing the thinking

$$(10+t)\frac{dP}{dt}+P=2(10+t) \quad \rightarrow \quad \frac{d}{dt}[(10+t)P]=2(10+t)$$

- Equation in form intended to suggest that it was the derivative of a product.
- Objective: assist students in discovering "the reverse product rule" (aka, the integrating factor approach)

Scene 2 class transcript

"Where have you seen this before?"

T: I was thinking initially, like, to me it looked like a chain rule kind of thing.

G: Chain rule?

T: But I couldn't get anywhere with that, though.

R: Yeah.

D: Differentiation by parts?

Scene 2 class transcript

G: Differentiation by parts? What is that?
D: I don’t remember, because …
C: Oh!
Ss: [laughter]
G: Craig, you said, “Oh.”
C: Well, yeah, somebody said the chain rule, and that, isn’t that like, ‘cause we got a derivative of the …
G: The chain works with an inner and outer function, right?
C: Yeah.
G: A function inside of another function. I don’t know. Do we have a function inside of another function here?

Scene 2 class transcript

The discussion continues:

G: The chain rule looks like what?

R: You multiply.

G: The chain rule … well, where do I go? Maybe here? If I become neutral and start using neutral letters, you know what? It's kind of like, if you have f of g of x, then if you differentiate you get f-primed evaluated at g of x times g-primed of x.

C: // Isn't the- //

G: // So on both sides // you really are looking at a chain, right? Something inside of some other function.

And then…

C: Is it the product rule?

T: I meant the product rule. That's what I was thinking.

G: Product rule!

Scene 2: Alternatives

- Both suggestions give something that can be "undone" by integrating
- "Differentiation by parts" is actually a pretty good answer
  - Could have asked student to elaborate or clarify
  - That might have brought up ideas that were more obviously productive
  - At his table, the student said, "like udv and vdu and that whole deal"
- Could have done more with "chain rule" than provide description of the rule

Scene 2: Interview

Gage discussed the students' "chain rule" suggestion:

"It's like 20 people in there and 18 of them say 'chain rule' when they mean 'product rule,' and I'm like, well, then we can't communicate."

"When they started talking about the chain rule, that just put a bullet through my head. I was like, whoa! Where does that come from? And what am I going to do with that?"

Scene 2: Summary

To make use of the “differentiation by parts” or “chain rule” suggestions, Gage would have needed to

- mine suggestions and follow students’ ideas

{*Component practice 1*}

- see mathematically useful connections between suggestions and desired answer

{*Component practice 2*}

Instead, Gage deployed his mathematical content knowledge to correct a perceived error. He attributed the failed discussion to a communication issue-- but they were closer than he thought.

Scene 3: (Not) Seeing students' challenges

A brief return to the population problem:

This situation can be modeled with a rate of change equation:

$$\frac{dP}{dt} = something$$

What should the "*something*" be? Should the rate of change be stated in terms of just *P*, just *t*, of both *P* and *t*? Make a conjecture about the right hand side of the rate of change equation and provide reasons for your conjecture."

Scene 3: (Not) Seeing students' challenges

- Some student responses:
  - "Anything that's a function of *P*, assuming *P* is a function of *t*, can be written as a function of t."
  - "If *P* was represented as a function of *t*, then wouldn't it change with a different initial population? Because, if we were saying that the rate of change is dependent only on the population, then that would be shifting the graph back and forth. "
- Conversation continued surrounding the role of "the initial population."

Scene 3: (Not) Seeing students' challenges

- Gage interrupts:

> "Can, can I ask, at least, I mean, we are trying to make a decision kind of, you know, what should be right here in this equation, and right now, I think we're kind of a little bit further down the road, you know. We're kind of, *P* of *t*, and initial populations, and, and, I'm not saying it's-. I'd like to maybe hear a couple other people's comments on it. Matt?"

Scene 3: Summary

- Although Gage knew that the initial conditions were as yet not relevant and "further down the road," *the students did not*. Understanding the irrelevance of the initial condition was central to the problem.
- In order to realize the significance of the students' discussion, Gage would have had to
  - Know that the irrelevance of the initial conditions would be a conceptual challenge for the students
  - Understand not just the *mathematical* point of the problem, but its pedagogical role in confronting a known conceptual challenge

Scene 4: Pursuing unproductive ideas

- Occurred just prior to "differentiation by parts" episode
- Students are considering how to integrate the expression
  - Matt says, "Ah, I get it!"
  - Matt's explanation is hard to follow but does not contain hints of product rule
  - Gage does not evaluate Matt's idea but asks to "hear one more" idea
  - Other students try to explain Matt's idea

Scene 4 transcript

"You may want to pursue this further, so that maybe it becomes a little clearer. … I'm not rejecting this. …outside [of class] then you pursue this a little further and see, and see what happens."

Scene 4 interview

"I just don't understand and haven't thought enough about differential equations as a subject to be taught so that I feel any flexibility at all. I mean I just feel like no matter what anybody says, I just don't know, 'Well, should I stop them because I don't know where this is going?' Well then I have to stop everybody essentially, and I have to go back to just telling them what I'm thinking. I just can't do it."

Scene 4: Summary

- Because he did not follow Matt's suggestion, Gage encouraged the pursuit of some mathematically unproductive ideas.
- And his approach did not help to illuminate the core ideas relevant to the discussion
- A different outcome would have required Gage to see/figure out the ideas were unproductive and either
  - direct attention away from the suggestion, or
  - pursue the suggestion so the ideas could be illuminated and understood by the class

Findings

- Being able to recognize or figure out that students' contributions are (not) potentially useful requires:
  - curriculum-specific PCK
  - specialized content knowledge
  - common content knowledge
- Understanding something "as a subject to be taught" takes effort and experience.

Conclusions

- Prof. Gage kept the discussions moving, but not always towards the mathematical goal
- He recognized this at a general level, but didn't always see the specific cause
- The specific cause: Challenges he faced in recognizing student contributions as *potentially* useful
- Findings corroborate other researchers' claims that "content knowledge is not enough"

Implications

- Teachers need to understand something "as a subject to be taught."
- We need to understand more about the knowledge needed to support various instructional practices.
- We need to design professional development opportunities to help teachers learn **all** the types of knowledge needed for teaching.

Contact Info + References

Natasha Speer
speer@math.umaine.edu

Joseph F. Wagner
wagner@xavier.edu

- Our earlier study:

  - Wagner, J., Speer, N., & Rossa, B. (2007). Beyond mathematical content knowledge: A mathematician's knowledge needed for teaching an inquiry-oriented differential equations course. *Journal of Mathematical Behavior*, *26*, 247-266.

- More recent work:

  - Speer, N. & Wagner, J. (in press). Knowledge needed by a teacher to provide analytic scaffolding during undergraduate mathematics classroom discussions, *Journal for Research in Mathematics Education*

References

- Ball, D. L., & Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics. In J. Boaler (Ed.), *Multiple perspectives on the teaching and learning of mathematics*. Westport, CT: Ablex.
- Begle, E. G. (1979). *Critical variables in mathematics education: Findings from a survey of the empirical literature*. Washington, DC: Mathematical Association of American and National Council of Teachers of Mathematics.
- Carpenter, T. P., Fennema, E., Peterson, P. L., & Carey, D. A. (1988). Teachers' pedagogical content knowledge of students' problem solving in elementary arithmetic. *Journal for Research in Mathematics Education*, *19*(5), 385–401.
- Carpenter, T. P., Fennema, E., Peterson, P. L., Chiang, C.-P., & Loef, M. (1989). Using knowledge of children's mathematics thinking in classroom teaching: An experimental study. *American Educational Research Journal*, *26*(4), 499–531.
- Fennema, E., Carpenter, T., Franke, M., Levi, L., Jacobs, V. R., & Empson, S. B. (1996). A longitudinal study of learning to use children's thinking in mathematics Instruction. *Journal for Research in Mathematics Education, 27*(4), 403-434.
- Hill, H., Rowan, B., & Ball, D. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*(2), 371-406.
- Hill, H., Schilling, S., & Ball, D. (2004). Developing measures of teachers' mathematics knowledge for teaching. *The Elementary School Journal, 105*(1), 11-30.
- Ma, L. (1999). *Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States*. Mahwah, NJ: Lawrence Erlbaum Assoc.
- Monk, D. (1994). Subject area preparation of secondary mathematics and science teachers and student achievement. *Economics of Education Review, 13*(2), 125-145.
- Wilson, S. M., Floden, R. E., & Ferrini-Mundy, J. (2002). Teacher preparation research: An insider's view from the outside. *Journal of Teacher Education, 53,* 190-204.
- Williams, S. R. & Baxter, J. A. (1996). Dilemmas of discourse-oriented teaching in one middle school mathematics classroom. *The Elementary School Journal, 97*(1), 21-38.