Relations Between Mathematics and Partner Disciplines

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Since Most of Our Students are from Other Fields:

- What should we know about them?
  - What other departments want us to do, topics, applications, examples
  - What pedagogy works for students in other fields
- How do we learn this?

- We need to learn their perspective on mathematics
  - Asymmetry in backgrounds

MAA: Curriculum Foundations Project

Reports to MAA from faculty in other disciplines on what they would like to see in mathematics courses their students take during the first two years of college:

"Voices from the Partner Disciplines"

www.maa.org/cupm/crafty/

organized by William Barker and Susan Ganter

Recurring Themes in Reports from Partner Disciplines

Need for

- Comfort with symbols and graphs as a language.
- Conceptual understanding.
- Mathematical modeling; facility with applications.

While formal theoretical knowledge of the concepts and a tool bag of techniques and computational skills are desirable, the most important factor is that students gain enough active understanding that they are able to think through and solve a wide variety of problems involving the fundamental concepts in a wide variety of contexts.

--*Physics*

Students must be able to follow and apply algebraic arguments, that is,

"listen to the equations,“

if they are to understand the relationships between various mathematical expressions, to adapt these expressions to particular applications, and to see that most specific mathematical expressions can be recovered from a few fundamental relationships in a few steps.

—Chemistry

Skills Regarded as Essential by Most Partner Disciplines

- Basic algebraic skills
- Concept of function
- Graphical reasoning
- Approximation, estimation; scale and size
- Numerical methods

We believe that spreadsheets should be used in the delivery of quantitative material to the fullest extent. Spreadsheets are very useful in charting data, conducting exploratory data analysis, developing models, demonstrating the impact of changes in the inputs on the outputs, carrying out parametric analysis, as well as more sophisticated applications such as optimization and simulation.

— *Business*

Spreadsheets advocated by:

- physics
- chemistry
- biology
- health/life sciences
- business
- civil engineering
- mechanical engineering
- chemical engineering
- biotechnology
- electronics
- information technology
- manufacturing

While much of the time in calculus courses is spent learning rules of differentiation and integration, what is more important for us is not that the students can take complicated derivatives, but rather that they are able to work with the abstract concept of "the derivative" and understand that it represents the slope, that if $u: R^2 \rightarrow R$, then $-u_1/u_2$ is the slope of a level surface of the function in x_1, x_2 space

Nolan Miller, Microeconomics, Harvard Kennedy School

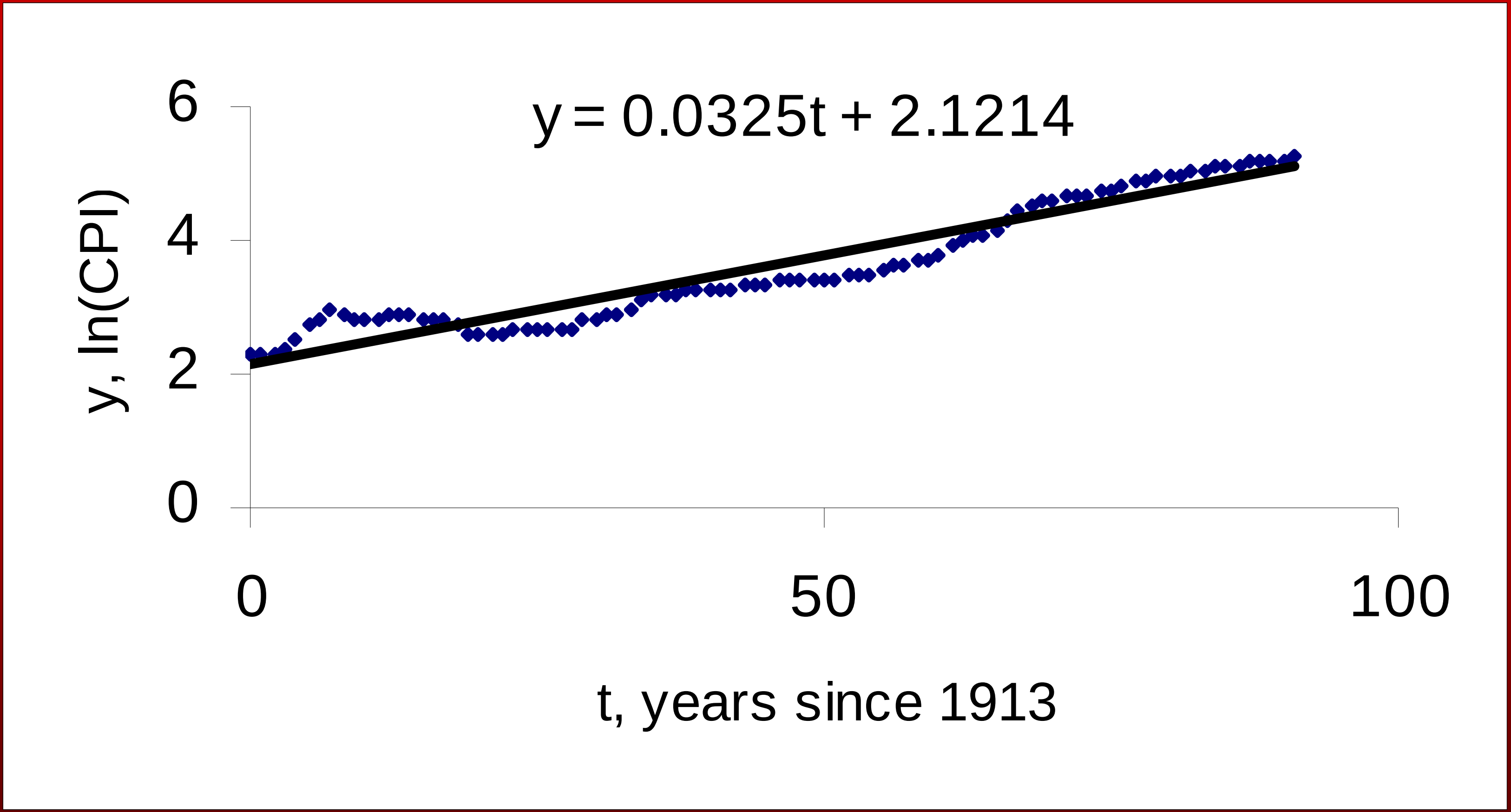
Economics: Opportunities for Misunderstandings

- Percentages and percentages points
- Derivative: *y' or y'/y*
- Logarithmic differentiation, eg for $P = K^{\alpha}L^{\beta}$
- Constrained optimization: use the Lagrangian or go directly to first order conditions; shadow price
- Difference equations vs. differential equations

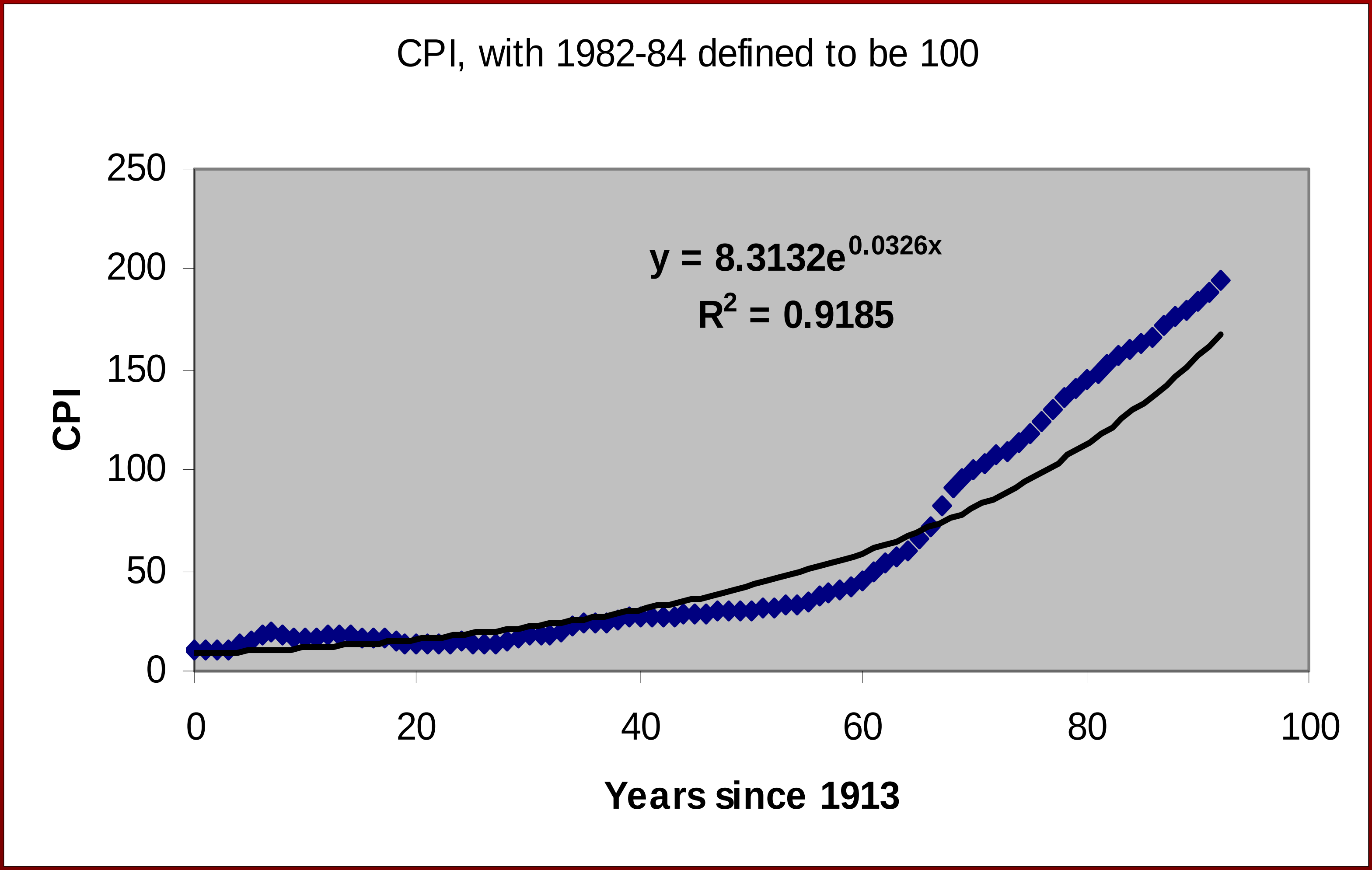
Things May Look Different "Over There"

Perspectives on Linearization: Economist

How fast has CPI grown over last century?



Another View of Same CPI Data



Things Look Different "Over There"

Perspectives on Linearization: Biologist

Michaelis-Menten Equation

- V_0 initial velocity of reaction
- $[S]_0$ is initial concentration of substrate
- V_{max}, K_M are constants

$$V_0 = \frac{V_{max}[S]_0}{K_m + [S]_0}$$

How Do We Help Students Transfer Knowledge?

To decide if a reaction follows Michaelis-Menten:

Linearize and plot data:

$$\frac{1}{V_0} = \frac{K_M + [S]_0}{V_{\max}[S]_0}$$

$$\frac{1}{V_0} = \frac{K_M}{V_{\max}} \cdot \frac{1}{[S]_0} + \frac{1}{V_{\max}}$$

But Some Things Aren’t So Different “Over There”

Another problem I've found is that students aren't good at struggling with a mathematical problem –– perhaps none of us are. We get students who have always done very well, and so they probably didn't have a lot of trouble with calculus. But, when they are faced with an abstract problem, they just don't know where to start.

Nolan Miller, Microeconomics,
Harvard Kennedy School

Data Example: Predicting Peak Oil Production

What is *peak oil production*?

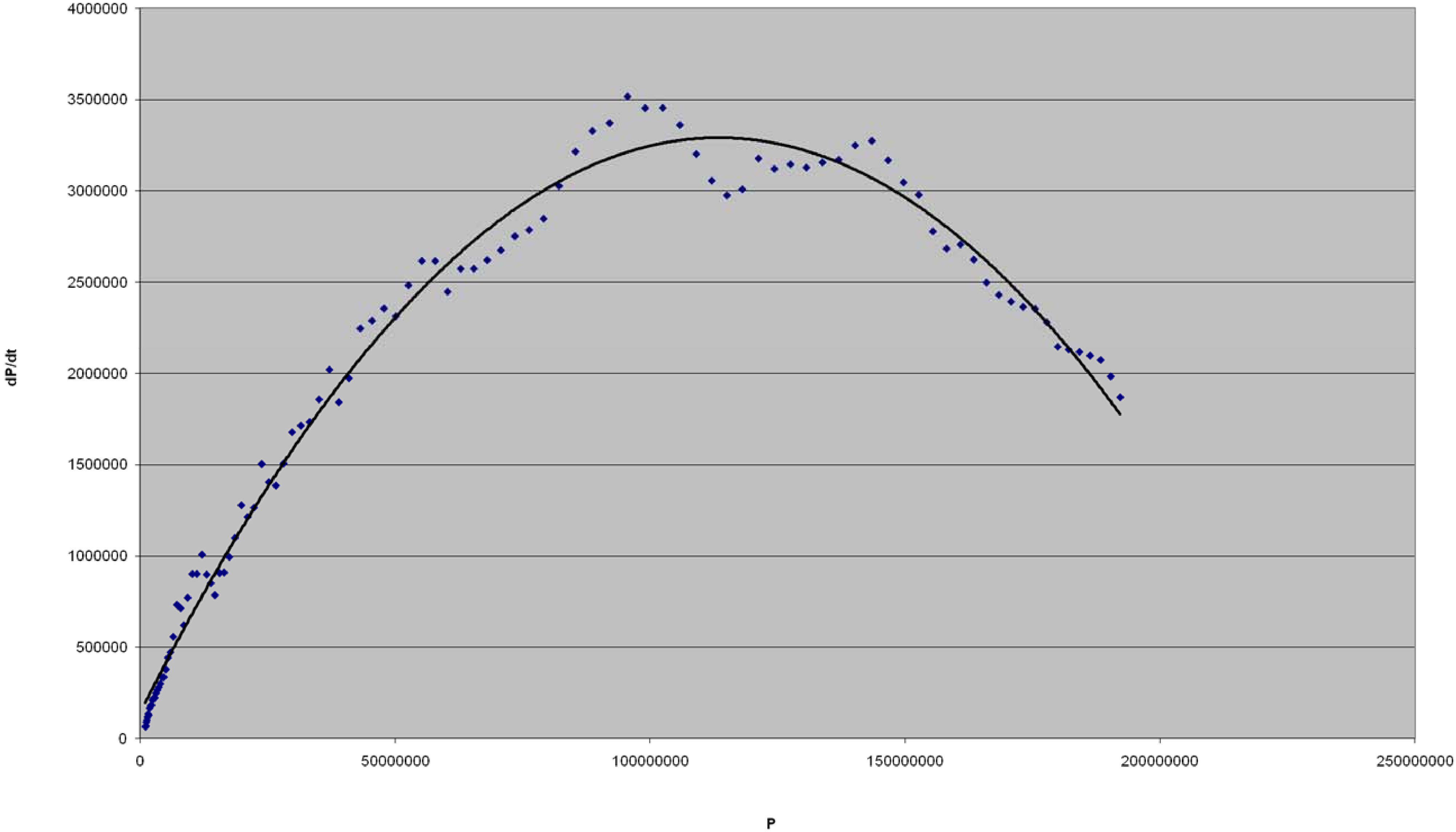
When will/did US oil production peak?

When will/did world oil production peak?

Why is this important? What are the economic implications?

(Analyze using 2008 data at www.eia.doe.gov)

US Oil Production (1900 to 2005) *dP/dt* vs. *P*
P is total oil produced in US since start of time



Total Oil Pumped in US since the Beginning of Time