

# The universal $sl(2)$ foam cohomology

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# The plan

- We have constructed a bi-graded cohomology theory for tangles that depends on two parameters  $h$  and  $a$ , which for the case of links, is a categorification of the classical Jones polynomial.

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- We have constructed a bi-graded cohomology theory for tangles that depends on two parameters  $h$  and  $a$ , which for the case of links, is a categorification of the classical Jones polynomial.
- This cohomology theory is properly functorial under link cobordisms, relative to boundary.
- For certain values of parameters  $a, h$  we obtain isomorphic versions of the Khovanov homology and Lee's variant of it.
- Therefore, we also obtain for free a version of the Khovanov homology which satisfies functoriality property with no sign ambiguity.

- Khovanov has explained how rank two Frobenius systems lead to homology theories, and showed that there is a universal one corresponding to a certain commutative Frobenius structure defined on  $\mathbb{Z}[X, a, h]/(X^2 - hX - a)$ .

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- Our construction is a geometric approach to this universal homology theory.

We need the condition that the ground ring contains the fourth-root of unity  $i$ . We also work with cobordisms with seams, as opposed to ordinary cobordisms.

- Let  $R := \mathbb{Z}[i][a, h]$ .
- We introduce a grading on the ring  $R$ :

$$\deg(h) = 2, \deg(a) = 4$$

$$\deg(1) = \deg(i) = 0.$$

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- Let  $\mathcal{A} = R[X]/(X^2 - hX - a)$  be the free  $R$ -module with generators  $\mathbf{1}$  and  $\mathbf{X}$ .
- We introduce a grading on  $\mathcal{A}$  by letting:

$$\deg(\mathbf{1}) = -1, \deg(\mathbf{X}) = 1.$$

$\mathcal{A}$  is commutative Frobenius with:

- multiplication

$$m : \mathcal{A} \otimes \mathcal{A} \rightarrow \mathcal{A}, \begin{cases} m(\mathbf{1} \otimes \mathbf{X}) = \mathbf{X} & m(\mathbf{1} \otimes \mathbf{1}) = \mathbf{1} \\ m(\mathbf{X} \otimes \mathbf{1}) = \mathbf{X} & m(\mathbf{X} \otimes \mathbf{X}) = h\mathbf{X} + a \end{cases}$$

- comultiplication

$$\Delta : \mathcal{A} \rightarrow \mathcal{A} \otimes \mathcal{A}, \begin{cases} \Delta(\mathbf{1}) = \mathbf{1} \otimes \mathbf{X} + \mathbf{X} \otimes \mathbf{1} - h\mathbf{1} \otimes \mathbf{1} \\ \Delta(\mathbf{X}) = \mathbf{X} \otimes \mathbf{X} + a\mathbf{1} \otimes \mathbf{1} \end{cases}$$

- unit  $\iota : R \rightarrow \mathcal{A}$ ,  $\iota(1) = \mathbf{1}$
- counit  $\epsilon : \mathcal{A} \rightarrow R$ ,  $\epsilon(\mathbf{1}) = 0$ ,  $\epsilon(\mathbf{X}) = 1$ .



# Computing the quantum $sl(2)$ -link invariant via webs

- The quantum  $sl(2)$ -link invariant is determined by the rules:

$$\begin{array}{c} \nearrow \\ \times \\ \searrow \end{array} = q \begin{array}{c} \curvearrowright \\ \quad \quad \end{array} \begin{array}{c} \curvearrowleft \\ \quad \quad \end{array} - q^2 \begin{array}{c} \searrow \\ \times \\ \nearrow \end{array}$$

$$\begin{array}{c} \searrow \\ \times \\ \nearrow \end{array} = q^{-1} \begin{array}{c} \curvearrowright \\ \quad \quad \end{array} \begin{array}{c} \curvearrowleft \\ \quad \quad \end{array} - q^{-2} \begin{array}{c} \nearrow \\ \times \\ \searrow \end{array}$$

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- This sends an oriented link diagram to a  $\mathbb{Z}[q, q^{-1}]$ -linear combination of disjoint union of *webs*.
- Webs** are circle graphs with bivalent vertices such that edges incident to a vertex are either both entering or both leaving the vertex.

## The web space

- Webs are evaluated via the relations:

$$\langle \bigcirc \cup \Gamma \rangle = (q + q^{-1}) \langle \Gamma \rangle = \langle \bigcirc \cup \Gamma \rangle$$

$$\langle \begin{array}{c} \swarrow \\ \searrow \end{array} \rangle = \langle \curvearrowright \rangle \quad \text{and} \quad \langle \begin{array}{c} \swarrow \\ \nearrow \end{array} \rangle = \langle \curvearrowleft \rangle$$

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- By these rules, a web with  $k$  connected components evaluates to  $(q + q^{-1})^k$ .
- The web space gives another way of computing the  $sl(2)$ -link invariant.

## A method for categorification

To categorify the  $sl(2)$ -link invariant we want to associate complexes to links instead of linear combinations.

Thus we want to replace a linear combination like

$$\begin{array}{c} \nearrow \\ \searrow \\ \swarrow \\ \searrow \end{array} = q^{-1} \left( \begin{array}{c} \nearrow \\ \searrow \end{array} \right) \left( - q^{-2} \begin{array}{c} \searrow \\ \nearrow \\ \swarrow \\ \searrow \end{array} \right)$$

with some complex of the form

$$\begin{array}{c} \nearrow \\ \searrow \\ \swarrow \\ \searrow \end{array} = \left[ 0 \longrightarrow q^{-1} \left( \begin{array}{c} \nearrow \\ \searrow \end{array} \right) \xrightarrow{?} q^{-2} \begin{array}{c} \searrow \\ \nearrow \\ \swarrow \\ \searrow \end{array} \longrightarrow 0 \right]$$

# Constructing complexes from link diagrams

Starting with a generic link diagram  $D$ , we associate to it a formal complex  $[D]$  whose construction is explained by:

$$\begin{array}{c}
 \begin{array}{c} \diagup \\ \diagdown \\ \diagdown \\ \diagup \end{array} = \left[ 0 \longrightarrow \begin{array}{c} \diagdown \\ \diagup \\ \diagup \\ \diagdown \end{array} \xrightarrow{\{2\}} \begin{array}{c} \text{web with red curve} \\ \text{dashed lines} \end{array} \xrightarrow{0} \begin{array}{c} \diagup \\ \diagdown \end{array} \xrightarrow{\{1\}} 0 \right] \\
 \qquad \qquad \qquad -1 \qquad \qquad \qquad 0
 \end{array}$$

$$\begin{array}{c}
 \begin{array}{c} \diagup \\ \diagdown \\ \diagup \\ \diagdown \end{array} = \left[ 0 \longrightarrow \begin{array}{c} \diagdown \\ \diagup \end{array} \xrightarrow{\{-1\}} \begin{array}{c} \text{web with red curve} \\ \text{dashed lines} \end{array} \xrightarrow{1} \begin{array}{c} \diagdown \\ \diagup \\ \diagup \\ \diagdown \end{array} \xrightarrow{\{-2\}} 0 \right] \\
 \qquad \qquad \qquad 0 \qquad \qquad \qquad 1
 \end{array}$$

## Constructing complexes from link diagrams

- The “chain objects” of  $[D]$  are column vectors of *webs* and “differentials” are matrices of *foams*.
- A **foam** is a cobordism between webs, and contains seams where orientations disagree.  
Specifically, the two facets incident with a seam have opposite orientations, which induces the same orientation on that seam.

## Constructing complexes from link diagrams

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- A **foam** is a cobordism between webs, and contains seams where orientations disagree.  
Specifically, the two facets incident with a seam have opposite orientations, which induces the same orientation on that seam.
- There is an ordering of the facets sharing a seam.
- Foams can have dots that are allowed to move freely along a facet they belong to, but can't cross seams.
- We read foams (as morphisms) from bottom to top, by convention, and we compose them by placing one on top of the other.



## The category **Foams**

- Let **Foams** be the category whose objects are webs and whose morphisms are foams, regarded up to boundary-preserving isotopies.
- We introduce a set of relations  $\ell$  among foams and denote the corresponding category by **Foams** $_{/\ell}$ .
- These relations are necessary in order to obtain a complex that is a link invariant (already) at the geometric level.

## Local relations

$$(2D) \quad \begin{array}{|c|} \hline \bullet \bullet \\ \hline \end{array} = h \begin{array}{|c|} \hline \bullet \\ \hline \end{array} + a \begin{array}{|c|} \hline \\ \hline \end{array}$$

(2D) is the geometric counterpart of relation in  $\mathcal{A}$ :

$$X^2 - hX - a = 0$$

## Local relations

$$(2D) \quad \text{parallelogram with two dots} = h \text{ parallelogram with one dot} + a \text{ empty parallelogram}$$

(2D) is the geometric counterpart of relation in  $\mathcal{A}$ :

$$X^2 - hX - a = 0$$

$$(CN) \quad \text{cylinder} = \text{cup with dot on top} + \text{cup with dot on bottom} - h \text{ empty cup}$$

The surgery formula (CN) is the geometric counterpart of:

$$\Delta(1) = 1 \otimes X + X \otimes 1 - h1 \otimes 1$$

## Local relations

### Closed foam relations

$$(S) \quad \begin{array}{c} \text{---} \\ \circ \\ \text{---} \end{array} = 0, \quad \begin{array}{c} \bullet \\ \circ \\ \text{---} \end{array} = 1$$

These relations correspond to  $\epsilon(1) = 0$ , and  $\epsilon(X) = 1$ , respectively.

## Local relations

### Closed foam relations

$$(S) \quad \begin{array}{c} \text{Sphere} \\ \text{with equator} \end{array} = 0, \quad \begin{array}{c} \text{Sphere} \\ \text{with equator and a dot} \end{array} = 1$$

These relations correspond to  $\epsilon(1) = 0$ , and  $\epsilon(X) = 1$ , respectively.

$$(F) \quad \begin{array}{c} \text{Foam with boundary} \\ \text{edges labeled 1 and 2} \end{array} = 0 = \begin{array}{c} \text{Foam with boundary} \\ \text{edges labeled 1 and 2, and a dot} \end{array}$$

$$\begin{array}{c} \text{Foam with boundary} \\ \text{edges labeled 1 and 2, and a dot} \end{array} = i = - \begin{array}{c} \text{Foam with boundary} \\ \text{edges labeled 1 and 2, and a dot} \end{array}$$

- The cutting neck relation (CN) implies the genus reduction formula:

$$\text{Genus-1 surface on base} = 2 \cdot \text{Square with dot} - h \cdot \text{Empty square}$$

- The cutting neck relation (CN) implies the genus reduction formula:

$$\text{Genus-1 surface on base} = 2 \text{ (square with dot)} - h \text{ (empty square)}$$

- By using the local relations  $\ell$ , we assign to any closed foam a unique element in  $R$ .  
 In particular we have:

$$\text{Foam with 1 loop} = 2, \quad \text{Foam with 2 loops} = 0, \quad \text{Foam with 3 loops} = 2h^2 + 8a.$$

## Both categories **Foams** and **Foams** $_{/\ell}$ are graded by degree

- We introduce a grading on the category **Foams** by defining the **degree** of a foam  $F$  with  $d$  dots:  $\deg(F) = -\chi(S) + 2d$ , where  $\chi$  is the Euler characteristic.

$$\deg \left( \text{foam with 0 dots} \right) = \deg \left( \text{foam with 1 dot} \right) = -1$$

$$\deg \left( \text{foam with 1 dot} \right) = \deg \left( \text{foam with 2 dots} \right) = 1$$

- The local relations  $\ell$  are degree-preserving, so **Foams** $_{/\ell}$  is also graded.
- Differentials in  $[D]$  are degree-preserving.



## Invariance and functoriality

### Theorem

*The formal complex  $[D]$  is invariant under the Reidemeister moves, up to homotopy.*

### Theorem

*This theory is **properly** functorial under link cobordisms; that is, given a 4-dimensional cobordism  $C$  between two links  $L_1$  and  $L_2$ , there is a well defined homomorphism  $[C] : [L_1] \rightarrow [L_2]$  between the corresponding formal complexes.*

## Some isomorphisms

The following relations hold in the category **Foams** $_{/e}$ :

$$\begin{array}{ccc}
 \begin{array}{c} \text{Diagram 1: A square with four arrows pointing outwards. Two vertical red lines are inside. The left line has an upward arrow labeled '1', and the right line has a downward arrow labeled '1'. The space between them is labeled '2' at the top and bottom.} \\ \end{array} = -i \begin{array}{c} \text{Diagram 2: A square with four arrows pointing outwards. Two red arcs are inside. The top arc has a downward arrow labeled '2', and the bottom arc has an upward arrow labeled '2'. The space between them is labeled '1' in the center.} \\ \end{array} \quad \text{and} \quad \begin{array}{c} \text{Diagram 3: A square with four arrows pointing outwards. Two vertical red lines are inside. The left line has a downward arrow labeled '2', and the right line has an upward arrow labeled '2'. The space between them is labeled '1' at the top and bottom.} \\ \end{array} = i \begin{array}{c} \text{Diagram 4: A square with four arrows pointing outwards. Two red arcs are inside. The top arc has an upward arrow labeled '1', and the bottom arc has a downward arrow labeled '1'. The space between them is labeled '2' in the center.} \\ \end{array}
 \end{array}$$
  

$$\begin{array}{ccc}
 \begin{array}{c} \text{Diagram 5: A square with four arrows pointing outwards. A red circle is inside. The left side of the circle has an upward arrow labeled '2', and the right side has a downward arrow labeled '1'.} \\ \end{array} = -i \begin{array}{c} \text{Diagram 6: A square with four arrows pointing outwards.} \\ \end{array} \quad \text{and} \quad \begin{array}{c} \text{Diagram 7: A square with four arrows pointing outwards. A red circle is inside. The left side of the circle has a downward arrow labeled '1', and the right side has an upward arrow labeled '2'.} \\ \end{array} = i \begin{array}{c} \text{Diagram 8: A square with four arrows pointing outwards.} \\ \end{array}
 \end{array}$$

which imply the isomorphisms below:

$$\begin{array}{ccc}
 \begin{array}{c} \text{Diagram 9: A square with four arrows pointing outwards. A red arc is inside, with a downward arrow labeled '1' on its left side.} \\ \end{array} \xleftrightarrow{\quad} \begin{array}{c} \text{Diagram 10: A square with four arrows pointing outwards. A red arc is inside, with an upward arrow labeled '1' on its left side.} \\ \end{array} \\
 \begin{array}{c} \text{Diagram 11: A square with four arrows pointing outwards. A red arc is inside, with a downward arrow labeled '1' on its left side.} \\ \end{array} \xleftrightarrow{-i} \begin{array}{c} \text{Diagram 12: A square with four arrows pointing outwards. A red arc is inside, with an upward arrow labeled '1' on its left side.} \\ \end{array}
 \end{array}
 \quad \text{and} \quad
 \begin{array}{ccc}
 \begin{array}{c} \text{Diagram 13: A square with four arrows pointing outwards. A red arc is inside, with a downward arrow labeled '1' on its left side.} \\ \end{array} \xleftrightarrow{\quad} \begin{array}{c} \text{Diagram 14: A square with four arrows pointing outwards. A red arc is inside, with an upward arrow labeled '1' on its left side.} \\ \end{array} \\
 \begin{array}{c} \text{Diagram 15: A square with four arrows pointing outwards. A red arc is inside, with a downward arrow labeled '1' on its left side.} \\ \end{array} \xleftrightarrow{i} \begin{array}{c} \text{Diagram 16: A square with four arrows pointing outwards. A red arc is inside, with an upward arrow labeled '1' on its left side.} \\ \end{array}
 \end{array}$$

## Passing to an algebraic category

We are now ready to apply a degree-preserving functor  $\mathcal{F}$  to pass to an algebraic category, and obtain an ordinary complex  $\mathcal{F}([D])$ :

$$\mathbf{Foams}_{/e} \xrightarrow{\mathcal{F}} R\text{-Mod}$$

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### Theorem

*The isomorphism class of the cohomology  $\mathcal{H}(D) := H(\mathcal{F}([D]))$  is a bi-graded invariant of  $L$ , whose graded Euler characteristic is the  $sl(2)$  invariant of  $L$ .*

## How is the functor $\mathcal{F}$ defined?

- Let  $\emptyset$  be the empty web.
- Define the functor  $\mathcal{F} : \mathbf{Foams}_{/\ell} \longrightarrow R\text{-Mod}$  as follows:

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on objects:  $\mathcal{F}(\Gamma) := \text{Mor}_{\mathbf{Foam}_{/\ell}}(\emptyset, \Gamma)$

on morphisms: Given  $S : \Gamma' \rightarrow \Gamma''$  define

$$\mathcal{F}(S) : \begin{array}{ccc} \text{Mor}(\emptyset, \Gamma') & \rightarrow & \text{Mor}(\emptyset, \Gamma'') \text{ by} \\ U & \mapsto & S \circ U \end{array}$$

## $\mathcal{F}(\Gamma)$ mimics the web space skein relations

We have:

- $\mathcal{F}(\emptyset) = R$
- $\mathcal{F}(\bigcirc) = \langle \text{web with top arrow}, \text{web with top arrow and dot} \rangle_R \cong \mathcal{A}$
- $\mathcal{F}(\text{web with two arrows}) = \langle \text{web with two arrows and dot}, \text{web with two arrows and dot} \rangle_R \cong \mathcal{A}$
- $\mathcal{F}(\Gamma \cup \bigcirc) \cong \mathcal{F}(\Gamma) \otimes_R \mathcal{A} \cong \mathcal{F}(\Gamma \cup \bigcirc)$
- $\mathcal{F}(\text{web with two arrows}) \cong \mathcal{F}(\text{web with one arrow}), \quad \mathcal{F}(\text{web with two arrows}) \cong \mathcal{F}(\text{web with one arrow})$

Therefore  $\mathcal{F}(\Gamma)$  is a free abelian group of graded rank  $\langle \Gamma \rangle$ .



## The invariant of a surface-knot

- Our theory is functorial under link cobordisms.
- A **surface-knot** (or **surface-link**)  $S$  is a closed surface embedded locally flatly in  $\mathbb{R}^4$ , and can be regarded as a cobordism from the empty link to itself.

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- A **surface-knot** (or **surface-link**)  $S$  is a closed surface embedded locally flatly in  $\mathbb{R}^4$ , and can be regarded as a cobordism from the empty link to itself.
- The induced map  $\mathcal{H}(\emptyset) \rightarrow \mathcal{H}(\emptyset)$  is an endomorphism of the ground ring  $R = \mathbb{Z}[i][a, h]$ , and yields an invariant of  $S$ , denoted here by  $\mathcal{L}(S)$ .
- The invariant  $\mathcal{L}(S)$  is determined only by the genus of  $S$ .

# The invariant of a surface-knot

## Theorem

*For any surface-knot  $S$  of genus  $g$ , we have the following:*

- 1 *If  $g$  is even, then  $\mathcal{L}(S) = 0$ ;*
- 2 *If  $g$  is odd, then  $\mathcal{L}(S) = 2(h^2 + 4a)^{\frac{g-1}{2}}$ .*

## Corollary

*For any torus-knot  $T^2$ , we have  $\mathcal{L}(T^2) = 2$ .*

## Working over $\mathbb{C}$

- Now let  $a, h \in \mathbb{C}$  and work over  $\mathbb{C}$ .
- There are two isomorphism classes of the corresponding link cohomology over  $\mathbb{C}$ , which are determined by the number of distinct roots of  $f(X) = X^2 - hX - a$ .

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- There are two isomorphism classes of the corresponding link cohomology over  $\mathbb{C}$ , which are determined by the number of distinct roots of  $f(X) = X^2 - hX - a$ .

(1) If  $h^2 + 4a = 0$ , thus  $f(X) = (X - \alpha)^2$  for some  $\alpha \in \mathbb{C}$ , there is an isomorphism between  $\mathcal{H}_{a,h}(L, \mathbb{C})$  and the original Khovanov homology over  $\mathbb{C}$ , induced by the isomorphism:

$$\begin{array}{|c|} \hline \bullet \\ \hline \end{array} \longrightarrow \begin{array}{|c|} \hline \bullet \\ \hline \end{array} - \alpha \begin{array}{|c|} \hline \\ \hline \end{array}$$

## Working over $\mathbb{C}$

(2) If  $f(X) = (X - \alpha)(X - \beta)$ , for some  $\alpha, \beta \in \mathbb{C}$  with  $\alpha \neq \beta$ , we have:

### Theorem

For any link with  $n$  components,  $\dim_{\mathbb{C}} \mathcal{H}_{a,h}(L, \mathbb{C}) = 2^n$ , and to each map  $\psi : \{\text{components of } L\} \rightarrow \mathcal{S} = \{\alpha, \beta\}$ , there exists a non-zero element  $\exists h_{\psi} \in \mathcal{H}_{a,h}(L, \mathbb{C})$  which lies in the cohomological degree

$$-2 \sum_{(u_1, u_2) \in \mathcal{S} \times \mathcal{S}, u_1 \neq u_2} lk(\psi^{-1}(u_1), \psi^{-1}(u_2)).$$

All  $h_{\psi}$ 's generate  $\mathcal{H}(L, \mathbb{C})$ .

## Filtered theory

Considering  $a, h \in \mathbb{C}$  such that  $h^2 + 4a \neq 0$ , one loses the bi-grading and obtains a  $\mathbb{Z}$ -graded theory with a filtration in place of the second grading (polynomial grading).

### Theorem

*For each  $a, h \in \mathbb{C}$  such that  $h^2 + 4a \neq 0$ , there is a spectral sequence converging to  $\mathcal{H}_{a,h}(L, \mathbb{C})$  with  $E_1$  term isomorphic to Khovanov's invariant over  $\mathbb{C}$ .*

### Theorem

*The  $E_1$  and higher terms of this spectral sequence are link invariants.*

## Bibliography

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THANK YOU!